GeoProof: A user interface for formal proofs in geometry.

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1. Related work and motivations
2. A general presentation of GeoProof
3. Proof related features
Dynamic Geometry.
There are quite many dynamic geometry software:
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Dynamic Geometry.

But few can deal with proofs:
Baghera, Cabri Euclide, Cabri Geometer, CaR, Chypre, Cinderella, Déclic, Défi, Dr. Geo, Euclid, Euklid DynaGeo, Eukleides, Gava, GCLC, GeoExp, GeoFlash, Geogebra, GeoLabo, Geometria, Geometrix, Geometry Explorer, Geometry Tutor, GeoPlanW, GeoSpaceW, GEUP, GeoView, GEX, GRACE, iGeom, KGeo, KIG, Non-Euclid, Sketchpad, Trace en poche, XCas . . .
1 - Interactive proof systems using a base of lemmas

- Baghera,
- Cabri-Euclide,
- Chypre,
- Défi,
- Geometrix,
- Geometry Tutor
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2 - Interfaces for an ATP

  Cinderella  Probalistic method, no proof shown.
  GCLC     Implementation of the area method, Wu’s method and Groebner basis method.
  Geometry Explorer Implementation of the full angle method using prolog, and visualization of the proofs in a diagrammatic way.
  GEX/Geometer Implementation of the area method, of Wu’s method and of deductive database methods, visualization of statements and some visual proofs.
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GeoView  Uses GeoPlan and Pcoq to visualize statements.
GeoProof combines these features:

- dynamic geometry
- automatic theorem proving
- interactive theorem proving using a proof assistant (Coq)
What is a proof assistant?

- The correctness of a proof is decidable by definition.
- A proof assistant is a system to check that a proof is correct.
Motivations

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- We should have both the ability to make arbitrarily complex proofs and use a base of known lemmas.
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• There are facts than can not be visualized graphically and there are facts that are difficult to understand without being visualized.

• We should have both the ability to make arbitrarily complex proofs and use a base of known lemmas.

• The verification of the proofs by the proof assistant provides a very high level of confidence.
Automated theorem proving in geometry in Coq:

- the area method for euclidean plane geometry
- Gröbner basis method (Loïc Pottier)
A quick overview of GeoProof

A prototype:

- written using ocaml and lablgtk2,
- distributed under the GPL2 licence,
- multi-platform.

http://home.gna.org/geoproof/
Dynamic geometry features

• points, lines, circles, vectors, segments, intersections, perpendicular lines, perpendicular bisectors, angle bisectors. . .

• central symmetry, translation and axial symmetry

• traces

• text labels with dynamic parts:
  • measures of angles, distances and areas
  • properties tests (collinearity, orthogonality, . . .)

• layers

• Computations use arbitrary precision

• Input: XML

• Output: XML, natural language, SVG, PNG, BMP, Eukleides (latex), Coq

Missing features:

• loci and conics

• macros

• animations
Proof related features

1. Automatic proof using an embedded ATP
2. Automatic proof using Coq
3. Interactive proof using Coq
Automatic proof using the embedded ATP

We need to perform a translation from a theory based on circles, lines and points to a theory based only on points.
/ passing through $A$ and $B$ \( \mathcal{P}_1(l) = A \mathcal{P}_2(l) = B \)

/ parallel line to \( m \) passing through $A$ \( \mathcal{P}_1(l) = A \mathcal{P}_2(l) = P2_1 \)

/ perpendicular to \( m \) passing through $A$ \( \mathcal{P}_1(l) = A \mathcal{P}_2(l) = P2_1 \)

/ perpendicular bisector of $A$ and $B$ \( \mathcal{P}_1(l) = P1_1 \mathcal{P}_2(l) = P2_1 \)

\( c \) circle of center $O$ passing through $A$ \( \mathcal{O}(c) = O \mathcal{P}(c) = A \)

\( c \) circle passing through $A,B$ and $C$ \( \mathcal{O}(c) = O_c \mathcal{P}(c) = A \)

\( c \) circle whose diameter is $A \mathcal{B}$ \( \mathcal{O}(c) = O_c \mathcal{P}(c) = A \)
Point $P$ on line $l$ \( \text{collinear}(P, P_1(l), P_2(l)) \)

Intersection of $l_1$ and $l_2$
\( \text{collinear}(l, P_1(l_1), P_2(l_1)) \wedge \text{collinear}(l, P_1(l_2), P_2(l_2)) \wedge \neg \text{parallel}(P_1(l_1), P_2(l_1), P_1(l_2), P_2(l_2)) \)

Perpendicular bisector of $AB$
\( P_1(l)A = P_1(l)B \wedge P_2(l)A = P_2(l)B \wedge P_1(l) \neq P_2(l) \wedge A \neq B \)
- Choose the fact you want to check:

Hypothesis:

\[ ((\text{is midpoint}(D, C)) \land \text{is midpoint}(E, C)) \land \neg C = A) \land A \land B \land C \land \neg D = E) \land \neg A = B \]

Conclusion:

\[ \text{parallel}(D, E, A, B) \]

- Choose the method you want to use:

- Groebner basis method
- Wu method
- Other Method

- Get the result:

The theorem is true
Dealing with non-degeneracy conditions

- Non degeneracy conditions play a crucial role in formal geometry.
- GeoProof allows to build a formula not a model of this formula.
- The user can define impossible figures.
-1- Choose the fact you want to check:
Hypothesis:
\{(is\_midpoint(C,A,B) \land is\_midpoint(D,B,A)) \land \lnot C = D\}

Conclusion:
true

-2- Choose the method you want to use:
- Gröbner bases method
- Wu method
- Chou method

-3- Get the result
The theorem is true (because the hypotheses are contradictory!)
Automatic proof using Coq

- Based on our formalization of the area method in Coq.
- Constructive theorems in euclidean plane geometry.
Require Export area_method.
Section Page_1.
Variable A:Point.
Variable B:Point.
Variable C:Point.
Variable D:Point.
Hypothesis HD: (is_midpoint D C A).
Variable E:Point.
Hypothesis HE: (is_midpoint E C B).
Goal (parallel E D B A).
Proof.
AutoGeom.
Qed.

AutoGeom.
Unnamed_thm is defined
Interactive proof using Coq

- GeoProof loads the library (axioms and theorems) and updates the interface.
- It translates each construction as an hypothesis in Coq syntax.
- It translates the conjecture into Coq syntax.
- It translates each construction into the application of a tactic to prove the existence of the newly introduced object.
Interactive proof using Coq

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Diagram:

```
Init → Construction → Goal Definition → Proof
```
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A possible framework for synchronization between GUIs
Another approach

Integrating the proof checker, the proving GUI, and the dynamic geometry software in a single window.
Future work

- Diagrammatic proofs
  - in geometry and
  - in abstract term rewriting.
- Tighter integration between the gui and proof assistant.
- Two ways communication between the proof assistant and the DGS.
My wishes

- A language/API to export/import statements.
- Statements should be relative to an axiom system.
- Statements should not impose a geometric construction.
- Non degeneracy conditions should not be overlooked.


Jürgen Richter-Gebert and Ulrich Kortenkamp.
Die interaktive geometrie software cinderella book and cd-rom.

http://cinderella.de.

Jacob T. Schwartz.
Probabilistic algorithms for verification of polynomial identities.