When a GUI for dynamic geometry becomes an interactive proof language.

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A diagram can be seen as a high level description of a proof.

Gauss (7-year-old):

\[ 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \]
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\[ 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \]
But sometimes a diagram can be misleading.

Original author unknown, this diagram is from Daniel Winterstein’s PhD.
Related work.

Diagrammatic Reasoning.

- The Diamond system (Mateja Jamnik’s phd)
- Dr Doodle (Daniel Winterstein’s phd)
- CDEG (Nathaniel Miller’s phd)
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Interactive Geometry.

But few can deal with proofs:

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Interactive geometry software are mainly used for educational purposes but few of them can deal with proofs.
Cinderella  Probabilistic method, no proof shown.

Geometrix  Interactive proof system using a base of lemmas.

Geometry Explorer  Implementation of the full angle method using prolog, and visualization of the proofs in a diagrammatic way.

GeoView  Uses GeoPlan and Pcoq to visualize statements.

GEX/Geometer  Implementation of the area method, of Wu’s method and of deductive database methods, visualization of statements only.
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My goal is to combine these features:

- dynamic geometry
- automation
- proof assistant
- interactive manual/diagrammatic proof
**Motivations**

- The use of a proof assistant provides a way to combine geometrical proofs with larger proofs (involving induction for instance).
- There are facts that cannot be visualized graphically and there are facts that are difficult to understand without a graphical approach.
- We should have both the ability to make arbitrarily complex proofs and use a base of known lemmas.
- The verification of the proofs by the proof assistant provides a high level of confidence.
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A quick overview of DrGeoCaml

- Based on Nicolas François’ work.
- Written using ocaml and lablgtk2.
- Distributed under the GPL2 licence.

https://gna.org/projects/geocaml
Drgeocaml’s features:
- points, lines, circles, vectors, segments, intersections, perpendicular lines, perpendicular bissectors, angle bissectors...
- central symmetry, translation, and axial symmetry
- text labels
- measures of angle, distances and areas

I/O: natural language, .kir, .zir, .csr, .svg, .png
- properties tests (collinearity, orthogonality, ...)
What about colors and style? should that be included in the proof language?
What is a diagram?

What is the difference between a diagram, a formula and a picture?

- There are elements of the diagram which do not have any meaning.
- A diagram is an example of something.
- A diagram is well defined.
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Generic Sketches

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Figure

A figure is a the couple formed of a set of points and a set of geometric constraints over these points.

Sketch

A sketch is a mapping from the set of points of a figure to $\mathbb{R}^2$ which is compatible with the constraints of that figure.

Generic Sketches (Informal Definition)

Given a figure $F$, we say that a set of sketches $G$ is generic for some property $P$ of arity $n$ iff

$$\forall (A_1, \ldots, A_n) \in F^n_{\text{points}}$$

$$(\forall s \in G, P_f(s(A_1), \ldots, s(A_n)) = \top) \Rightarrow (\forall \text{Pts}, F_C \rightarrow P(A_1, \ldots, A_n))$$
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Examples

Three free points.
- collinear: any non degenerated triangle is a generic sketch
- collinear and equal_length: non isosceles, non degenerated triangles are generic sketches

Three collinear points
Between \( \rightarrow \) Several Cases
Negative predicates

- Negative predicates are difficult to visualize.

Three points

\( \neq \rightarrow \) you need to put the three points at the same place.
Gui/Language

Unification
Proof of an existential theorem
Assert a new fact
Delete an assumption
Prove that the theorem is false

Diagram matching
Macro
Mark a fact on the diagram
Delete some object
Collect counter examples
Animation or Graph?

- The process of doing the diagram is important.
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• We do need to mix visualization, automation, and interactive proof.
• Visualizing negative statements is difficult.
• Diagrams have to come with a notion which tells what we can deduce from the diagram.
• We need animation.

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