Stage de Master Informatique (printemps/été 2026)
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Automatic proofs in projective geometry : finite projective spaces, spreads, and packings

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Projective geometry is an approach to geometry that captures the concepts of *perspective* and *horizon*. In 2D, this amounts to the assumption that any two lines always intersect. In 3D, this is equivalent to saying that any two coplanar lines always intersect.

Projective geometry can be modeled by a very simple system of axioms. In this context, it can be easily proven that certain finite spaces (containing a finite number of points and lines) satisfy the axioms of projective geometry [5, 1].

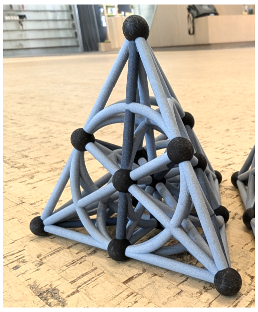

Figure 1 – A physical representation of PG(3,2)

In this work, we will focus on the smallest projective space of dimension 3 : PG(3,2). Fig. 1 gives a physical representation of this finite projective space[1] which contains 15 points and 35 lines. A spread is a partition of the points into disjoint lines. In PG(3,2), there are 56 different spreads (each consisting of 5 lines). All these spreads are isomorphic, which means that we can define a collineation (a bijection that respects the incidence relation) between 2 of these spreads. A packing of PG(3,2) is a partition of the 35 lines into 7 disjoint spreads each containing 5 lines. There are 240 packings divided into two classes of equal size in PG(3,2).

We formally described the projective space PG(3,2) in Coq/Rocq [4] by representing points and lines with simple inductive types, with a boolean incidence relation. We then defined and enumerated all collineations, spreads, and packings, as well as their classification. This case study relied on an automatic generation tool for most of the specifications as well as some proof scripts. However, it required the manual writing of several low-level proof scripts.

The objective of this internship is to study how to reproduce proofs made in Coq/Rocq using automatic SAT/SMT provers, such as Z3 [2]. Given the size of the proofs to be produced, we will need to consider breaking the problem down into subproblems that can be proven automatically by the system, and in particular study how to take advantage of the problem's numerous symmetries. We could also study how to perform these proofs directly in the Coq/Rocq proof assistant, using its SMTCoq extension [3].

1. from `https://en.wikipedia.org/wiki/PG(3,2)`

Références

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[3] Burak Ekici, Alain Mebsout, Cesare Tinelli, Chantal Keller, Guy Katz, Andrew Reynolds, and Clark W. Barrett. SMTCoq : A Plug-In for Integrating SMT Solvers into Coq. In Rupak Majumdar and Viktor Kuncak, editors, *Computer Aided Verification - 29th International Conference, CAV 2017, Heidelberg, Germany, July 24-28, 2017, Proceedings, Part II*, volume 10427 of *Lecture Notes in Computer Science*, pages 126–133. Springer, 2017.

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[5] Nicolas Magaud, Julien Narboux, and Pascal Schreck. Formalizing Projective Plane Geometry in Coq. In Thomas Sturm, editor, *Post-proceedings of ADG'2008*, volume 6301 of *LNAI*. Springer, 2008.