Stage de Master Informatique (printemps/été 2026)
Master Internship (spring/summer 2026)
Finite projective planes

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Projective geometry is an approach to geometry that captures the notions of *perspective* and *horizon*. In 2D, this means assuming that any 2 straight lines always intersect. Projective geometry can be modeled by a very simple system of axioms. In this case, we can easily prove that certain finite planes (containing a finite number of points and lines) do indeed verify the axioms of projective geometry [5]. This is the case, for example, of the Fano plane shown in the figure below.

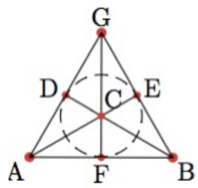

Figure 1: The Fano plane PG(2,2)

The Coq/Rocq system is a proof assistant dedicated to both mathematics and computer science [1]. In particular, it can be used to formally describe mathematical theories and to construct theorem provers based on these theories. It works interactively. Users interactively construct what they believe to be a proof of a theorem, and the system automatically checks that the constructed proof does indeed demonstrate the theorem in question.

The Coq/Rocq proof assistant has been successfully used to formally prove some interesting properties of finite models of projective geometry. In [5], we consider some small projective planes, e.g. Fano plane and prove that their formal description as an point-line incidence structure verifies the axioms of projective plane geometry. In [4], we then formalize a larger (3D) model : the smallest projective space PG(3,2), featuring 15 points and 35 lines, and some properties of its characteristic subsets, namely spreads and packings.

The goal of this internship is to formally describe in Coq/Rocq some projective planes of the shape PG(2,q), and study their existence and/or their unicity depending on q. Such combinatorial results include formally proving that there are only four non-isomorphic projective space of order 9 [3] (PG(2,9), Hall plane, its dual and Hughes plane) [1] and that there is no finite projective plane of order 10 [2]. We shall first reproduce these results which were initially obtained by computational means. The next step will be to describe the notions at stake, their properties and then to formally prove them in Coq/Rocq.

[1]Their descriptions are available at `https://ericmoorhouse.org/pub/planes/`.

References

[1] Yves Bertot and Pierre Castéran. *Interactive Theorem Proving and Program Development, Coq'Art: The Calculus of Inductive Constructions.* Springer, 2004.

[2] C. W. H. Lam, L. Thiel, and S. Swiercz. The non-existence of finite projective planes of order 10. *Canadian Journal of Mathematics*, 41(6):1117–1123, 1989.

[3] C.W.H. Lam, G. Kolesova, and L. Thiel. A computer search for finite projective planes of order 9. *Discrete Mathematics*, 92(1):187–195, 1991.

[4] Nicolas Magaud. Proof Pearl: Formalizing Spreads and Packings of the Smallest Projective Space PG(3, 2) Using the Coq Proof Assistant. In June Andronick and Leonardo de Moura, editors, *13th International Conference on Interactive Theorem Proving, ITP 2022, August 2022, Haifa, Israel*, volume 237 of *LIPIcs*, pages 25:1–25:17. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2022.

[5] Nicolas Magaud, Julien Narboux, and Pascal Schreck. Formalizing Projective Plane Geometry in Coq. In Thomas Sturm and Christoph Zengler, editors, *Automated Deduction in Geometry - 7th International Workshop, ADG 2008, Shanghai, China, September 22-24, 2008. Revised Papers*, volume 6301 of *Lecture Notes in Computer Science*, pages 141–162. Springer, 2008.