Scale-space features with mathematical morphology

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Outline

1. Introduction
2. Multiscale representation
3. 1-D features
4. 2-D features
5. Applications
6. Conclusion
Outline

1 Introduction

2 Multiscale representation

3 1-D features

4 2-D features

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6. Conclusion
Basics of Mathematical morphology

Definition

A framework for **analysis of spatial structures in images.**
A non-linear alternative to existing image processing toolboxes.
A set of operators applied on an image $f : E \rightarrow T$ with a pattern (SE) $b$.
Two fundamentals operators (erosion $\varepsilon$ and dilation $\delta$) . . .
. . . from which are built more complex ones.

Theoretical framework

For binary images, MM may be formalized with set theory . . .
but not for more complex images (grayscale, multispectral).
Thus MM is rather defined using **complete lattice theory.**
Basics of Mathematical morphology

Definition
A framework for **analysis of spatial structures in images**. A non-linear alternative to existing image processing toolboxes. A set of operators applied on an image \( f : E \rightarrow T \) with a pattern (SE) \( b \). Two fundamentals operators (erosion \( \varepsilon \) and dilation \( \delta \)) ... from which are built more complex ones.

Theoretical framework
For binary images, MM may be formalized with set theory ... but not for more complex images (grayscale, multispectral). Thus MM is rather defined using **complete lattice theory**.
## Basics of Mathematical morphology

### Complete lattice theory

A complete lattice is defined from:

- a partially **ordered set** \((T, \geq)\) (*e.g.* the natural order of scalars)
- an **infimum** or greatest lower bound \(\land\) (*e.g.* minimum)
- a **supremum** or least upper bound \(\lor\) (*e.g.* maximum)

### Main operators

<table>
<thead>
<tr>
<th>Operation</th>
<th>Formula</th>
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<tbody>
<tr>
<td>Erosion:</td>
<td>(\varepsilon_b(f)(p) = \land_{q \in b} f(p + q), \ p \in E)</td>
</tr>
<tr>
<td>Dilation:</td>
<td>(\delta_b(f)(p) = \lor_{q \in \check{b}} f(p + q) = \lor_{q \in b} f(p - q), \ p \in E)</td>
</tr>
<tr>
<td>Opening:</td>
<td>(\gamma_b(f) = \delta_b(\varepsilon_b(f)))</td>
</tr>
<tr>
<td>Closing:</td>
<td>(\varphi_b(f) = \varepsilon_b(\delta_b(f)))</td>
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Basics of Mathematical morphology

Complete lattice theory

A complete lattice is defined from:
- a partially **ordered set** \((T, \geq)\) *(e.g. the natural order of scalars)*
- an **infimum** or greatest lower bound \(\wedge\) *(e.g. minimum)*
- a **supremum** or least upper bound \(\vee\) *(e.g. maximum)*

Main operators

**Erosion:** \(\varepsilon_b(f)(p) = \bigwedge_{q \in b} f(p + q), \ p \in E\)
**Dilation:** \(\delta_b(f)(p) = \bigvee_{q \in b} f(p + q) = \bigvee_{q \in b} f(p - q), \ p \in E\)
**Opening:** \(\gamma_b(f) = \delta_b(\varepsilon_b(f))\)
**Closing:** \(\varphi_b(f) = \varepsilon_b(\delta_b(f))\)
On the choice of SE

- disc
- diamond
- square

- cross
- horizontal
- vertical

size $\lambda = 1$, $\lambda = 2$, $\lambda = 3$

size $\lambda = 1$, $\lambda = 2$, $\lambda = 3$
Exemple 1: binary case (erosion and dilation)

\[ \varepsilon_0(f) = f \]
\[ \varepsilon_1(f) \]
\[ \varepsilon_2(f) \]
\[ \varepsilon_3(f) \]
\[ \varepsilon_4(f) \]

\[ \delta_0(f) = f \]
\[ \delta_1(f) \]
\[ \delta_2(f) \]
\[ \delta_3(f) \]
\[ \delta_4(f) \]
Exemple 1: binary case (opening and closing)

\[ \gamma_0(f) = f \] 
\[ \gamma_1(f) \] 
\[ \gamma_2(f) \] 
\[ \gamma_3(f) \] 
\[ \gamma_4(f) \]

\[ \varphi_0(f) = f \] 
\[ \varphi_1(f) \] 
\[ \varphi_2(f) \] 
\[ \varphi_3(f) \] 
\[ \varphi_4(f) \]
Exemple 2: grayscale case (erosion and dilation)

\[ \varepsilon_0(f) = f \quad \varepsilon_2(f) \quad \varepsilon_4(f) \quad \varepsilon_6(f) \quad \varepsilon_8(f) \]

\[ \delta_0(f) = f \quad \delta_2(f) \quad \delta_4(f) \quad \delta_6(f) \quad \delta_8(f) \]
Exemple 2: grayscale case (opening and closing)

\[ \gamma_0(f) = f \quad \gamma_2(f) \quad \gamma_4(f) \quad \gamma_6(f) \quad \gamma_8(f) \]

\[ \varphi_0(f) = f \quad \varphi_2(f) \quad \varphi_4(f) \quad \varphi_6(f) \quad \varphi_8(f) \]
Algebraic filters: avoid the sensitivity to the SE

**Geodesic operators**

\[ \varepsilon_g^{(1)}(f)(p) = \varepsilon^{(1)}(f)(p) \lor g(p), \quad \varepsilon_g^{(n)}(f) = \varepsilon^{(1)}(\varepsilon_g^{(n-1)}(f)) \]

\[ \delta_g^{(1)}(f)(p) = \delta^{(1)}(f)(p) \land g(p), \quad \delta_g^{(n)}(f) = \delta^{(1)}(\delta_g^{(n-1)}(f)) \]

**Reconstruction operators**

\[ \rho_g^{\varepsilon}(f) = \varepsilon^{(j)}(f) \text{ with } j \text{ such as } \varepsilon^{(j)}(f) = \varepsilon^{(j-1)}(f) \]

\[ \rho_g^{\delta}(f) = \delta^{(j)}(f) \text{ with } j \text{ such as } \delta^{(j)}(f) = \delta^{(j-1)}(f) \]

**Filters by reconstruction**

\[ \gamma_b^{\rho}(f) = \rho_f^{\delta}(\varepsilon_b(f)) \]

\[ \varphi_b^{\rho}(f) = \rho_f^{\varepsilon}(\delta_b(f)) \]
**Algebraic filters: avoid the sensitivity to the SE**

**Filters applied with a family of SE**

\[
\gamma^\alpha_B(f) = \bigvee_{b \in B} \gamma_b(f) \\
\varphi^\alpha_B(f) = \bigwedge_{b \in B} \varphi_b(f)
\]

**Area filters**

\[
\gamma^a_\lambda(f) = \bigvee_b \{ \gamma_b(f) \mid b \text{ is connected and } \text{card}(b) = \lambda \} \\
\varphi^a_\lambda(f) = \bigwedge_b \{ \varphi_b(f) \mid b \text{ is connected and } \text{card}(b) = \lambda \}
\]

**Attribute filters**

\[
\gamma^\chi_\lambda(f) = \bigvee_b \{ \gamma_b(f) \mid b \text{ is connected and } \chi(b, \lambda) \} \\
\varphi^\chi_\lambda(f) = \bigwedge_b \{ \varphi_b(f) \mid b \text{ is connected and } \chi(b, \lambda) \}
\]

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Scale-space

Linear scale-space by means of a convolution with Gaussian kernel

\[ \Upsilon_t(f)(x, y) = f(x, y) * g(x, y, t) \]
\[ = \int_{-\infty}^{+\infty} f(u, v) \frac{1}{2\pi t^2} e^{-\frac{(x-u)^2+(y-v)^2}{2t^2}} \, du \, dv \]

Axioms

- **causality** no additional structure is created in the image (both height and position of extrema are preserved)
- **recursivity** \( \Upsilon_t(\Upsilon_s(f)) = \Upsilon_s(\Upsilon_t(f)) = \Upsilon_{t+s}(f) \), \( \forall t, s \geq 0 \)
- **increasingness** if \( f \leq g \), \( \Upsilon_t(f) < \Upsilon_t(g) \), \( \forall t > 0 \)
- **anti-extensivity** \( \Upsilon_t(f) \geq f \) so \( t_1 \leq t_2 \), \( \Upsilon_{t_1}(f) < \Upsilon_{t_2}(f) \)
- or **extensivity** \( \Upsilon_t(f) \leq f \) so \( t_1 \leq t_2 \), \( \Upsilon_{t_1}(f) > \Upsilon_{t_2}(f) \)
Morphological scale-spaces

Straightforward applications of MM
- using erosions or dilations
- using alternate sequential filters (ASF)
- using advanced representations (e.g. max-tree)

Recursivity in the morphological framework
The recursivity axiom is replaced by the absorption law:
$$\forall t, s \geq 0, \ \Gamma_t(\Gamma_s(f)) = \Gamma_s(\Gamma_t(f)) = \Gamma_{\text{max}(t,s)}(f)$$

A new axiom: idempotence
$$\Gamma_t(\Gamma_t(f)) = \Gamma_t(f)$$
### Gaussian vs. Morphological Scale-space

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<th>Multiscale representation</th>
<th>1-D features</th>
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Scale-space features with mathematical morphology
Gaussian vs. Morphological Scale-space
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**Gaussian vs. Morphological Scale-space**

![Gaussian vs. Morphological Scale-space](image)

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Scale-space features with mathematical morphology
Morphological series

Series of successive filtered images

\[ \Pi^\psi(f) = \left\{ \Pi^\psi_\lambda(f) \mid \Pi^\psi_\lambda(f) = \psi_\lambda(f) \right\}_{0 \leq \lambda \leq n} \]

with \( \psi \) a morphological filter (e.g. opening \( \gamma \) or closing \( \varphi \)) and \( \psi_\lambda \) a shortcut for \( \psi_{b\lambda} \).

Dual series

\[ \Pi(f) = \left\{ \Pi_\lambda(f) \mid \Pi_\lambda(f) = \begin{cases} \Pi^\varphi_\lambda(f), & \lambda < 0 \\ \Pi^\gamma_\lambda(f), & \lambda > 0 \\ f, & \lambda = 0 \end{cases} \right\}_{-n \leq \lambda \leq n} \]
Morphological series

Illustrative example using openings and closings by reconstruction

<table>
<thead>
<tr>
<th>Image 1</th>
<th>Image 2</th>
<th>Image 3</th>
<th>Image 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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<tr>
<td>Image 5</td>
<td>Image 6</td>
<td>Image 7</td>
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<tr>
<td>Image 9</td>
<td>Image 10</td>
<td>Image 11</td>
<td>Image 12</td>
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<tr>
<td>Image 13</td>
<td>Image 14</td>
<td>Image 15</td>
<td>Image 16</td>
</tr>
<tr>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>
Morphological differential series

Differential series of successive filtered images

\[ \Delta^\gamma(f) = \{ \Delta^\gamma_\lambda(f) \mid \Delta^\gamma_\lambda(f) = \Pi^\gamma_\lambda(f) - \Pi^\gamma_{\lambda-1}(f) \}_{0 \leq \lambda \leq n} \]

and

\[ \Delta^\varphi(f) = \{ \Delta^\varphi_\lambda(f) \mid \Delta^\varphi_\lambda(f) = \Pi^\varphi_\lambda(f) - \Pi^\varphi_{\lambda-1}(f) \}_{0 \leq \lambda \leq n} \]

Dual differential series

\[ \Delta(f) = \begin{cases} \Delta_\lambda(f) \mid \Delta_\lambda(f) = \begin{cases} \Delta^\varphi_{-\lambda}(f), & \lambda < 0 \\ \Delta^\gamma_\lambda(f), & \lambda > 0 \\ 0, & \lambda = 0 \end{cases} \end{cases} \] \quad -n \leq \lambda \leq n \]
Morphological differential series

<table>
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<tr>
<td><img src="image-url" alt="Image of morphological differential series" /></td>
</tr>
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</table>

Scale-space features with mathematical morphology
Other morphological series

Various series

- structural filters $\psi$
- filters by reconstruction $\psi^\rho$
- attribute filters $\psi^\chi$, in particular area filters $\psi^a$

but also . . .

- any morphological operator $\nu$ such as erosion $\varepsilon$ or dilation $\delta$
- morphological gradients $G_\lambda(f) = \delta_\lambda(f) - \varepsilon_\lambda(f)$
- top-hats $\tau^\gamma(f) = f - \gamma(f)$ and $\tau^\varphi(f) = \varphi(f) - f$
Differential Morphological Profile

Each pixel \( p \) is associated with \( \Pi^\psi(f)(p) = (\Pi^\psi_\lambda(f)(p))_{0 \leq \lambda \leq n} \). The DMP is defined by \( \text{DMP}(f)(p) = \Delta^\rho(f)(p) \).
Local features

Opening transform

For a binary image, each pixel $p$ is associated with the coarsest scale it appears:

$$\Xi_\psi(f)(p) = \max\{\lambda \geq 0 | \psi_\lambda(f)(p) > 0\}$$
Global features

Granulometry: successive openings

\[ \Omega^\gamma(f) = \left\{ \Omega^\gamma_\lambda(f) | \Omega^\gamma_\lambda(f) = \sum_{p \in E} \Pi^\gamma_\lambda(f)(p) \right\} \quad 0 \leq \lambda \leq n \]

Anti-granulometry: successive closings

\[ \Omega^\varphi(f) = \left\{ \Omega^\varphi_\lambda(f) | \Omega^\varphi_\lambda(f) = \sum_{p \in E} \Pi^\varphi_\lambda(f)(p) \right\} \quad 0 \leq \lambda \leq n \]

Towards a cumulative distribution function

\[ \Gamma^\psi(f) = \left\{ \Gamma^\psi_\lambda(f) | \Gamma^\psi_\lambda(f) = 1 - \frac{\Omega^\psi_\lambda(f)}{\Omega^\psi_0(f)} \right\} \quad 0 \leq \lambda \leq n \]
Global features

**Granulometry: successive openings**

\[
\Omega^\gamma(f) = \left\{ \Omega^\gamma_\lambda(f) \mid \Omega^\gamma_\lambda(f) = \sum_{p \in E} \Pi^\gamma_\lambda(f)(p) \right\}_{0 \leq \lambda \leq n}
\]

**Anti-granulometry: successive closings**

\[
\Omega^\phi(f) = \left\{ \Omega^\phi_\lambda(f) \mid \Omega^\phi_\lambda(f) = \sum_{p \in E} \Pi^\phi_\lambda(f)(p) \right\}_{0 \leq \lambda \leq n}
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**Towards a cumulative distribution function**

\[
\Gamma^\psi(f) = \left\{ \Gamma^\psi_\lambda(f) \mid \Gamma^\psi_\lambda(f) = 1 - \frac{\Omega^\psi_\lambda(f)}{\Omega^\psi_0(f)} \right\}_{0 \leq \lambda \leq n}
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Scale-space features with mathematical morphology
Global features

Granulometry: successive openings

$$\Omega^\gamma(f) = \left\{ \Omega_\lambda^\gamma(f) \mid \Omega_\lambda^\gamma(f) = \sum_{p \in E} \Pi_\lambda^\gamma(f)(p) \right\}$$

$$0 \leq \lambda \leq n$$

Anti-granulometry: successive closings

$$\Omega^\varphi(f) = \left\{ \Omega_\lambda^\varphi(f) \mid \Omega_\lambda^\varphi(f) = \sum_{p \in E} \Pi_\lambda^\varphi(f)(p) \right\}$$

$$0 \leq \lambda \leq n$$

Towards a cumulative distribution function

$$\Gamma^\psi(f) = \left\{ \Gamma_\lambda^\psi(f) \mid \Gamma_\lambda^\psi(f) = 1 - \frac{\Omega_\lambda^\psi(f)}{\Omega_0^\psi(f)} \right\}$$

$$0 \leq \lambda \leq n$$
Global features

Illustration of the granulometry / anti-granulometry

Granulometric curve

- Cross SE
- Diamond SE
- Disc SE
- Square SE

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**Global features**

**Pattern spectrum: size distribution**

\[
\Phi(f) = \left\{ \Phi_\lambda(f) \mid \Phi_\lambda(f) = \sum_{p \in E} \Delta_\lambda(f)(p) \right\}_{-n \leq \lambda \leq n}
\]

**Normalized distribution**

\[
\Lambda(f) = \left\{ \Lambda_\lambda(f) \mid \Lambda_\lambda(f) = \frac{\Phi_\lambda(f)}{\Omega_0(f)} \right\}_{-n \leq \lambda \leq n}
\]
Global features

Pattern spectrum: size distribution

\[ \Phi(f) = \left\{ \Phi_{\lambda}(f) \mid \Phi_{\lambda}(f) = \sum_{p \in E} \Delta_{\lambda}(f)(p) \right\}_{-n \leq \lambda \leq n} \]

Normalized distribution

\[ \Lambda(f) = \left\{ \Lambda_{\lambda}(f) \mid \Lambda_{\lambda}(f) = \frac{\Phi_{\lambda}(f)}{\Omega_{0}(f)} \right\}_{-n \leq \lambda \leq n} \]
Global features

Illustration of the pattern spectrum

Pattern Spectrum

- Structural filters
- Filters by reconstruction
- Area filters

SE size

Scale-space features with mathematical morphology
Global features

Pattern spectrum vs. histogram

Grayscale Histogram

Target squares
Small squares

Gray level

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Global features

Pattern spectrum vs. histogram

- Target squares
- Small squares

Pattern Spectrum

SE size

Scale-space features with mathematical morphology

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Global features

Morphological covariance: erosion series with a 2-pixel SE

\[ K_{\vec{v}}(f) = \left\{ K_{\lambda}^{\vec{v}}(f) \mid K_{\lambda}^{\vec{v}}(f) = \sum_{p \in E} \Pi_{\lambda, \vec{v}}(f)(p) \right\} \]

with \( \varepsilon_{\lambda, \vec{v}}(f)(p) = f(p - \lambda \vec{v}) \land f(p + \lambda \vec{v}) \)
Limitations of 1-D features

1-D features are limited to a single evolution curve. Their only parameter is the SE size. They cannot gather complementary information.

Various 2-D features

- size-shape
- size-orientation
- size-spectral
- size-intensity
- size-spatial
Size-shape

**One SE $b_{\alpha,\beta}$ with 2 different size parameters $\alpha$ and $\beta$**

\[
\Pi^\psi(f) = \left\{ \Pi^\psi_{\alpha,\beta}(f) \mid \Pi^\psi_{\alpha,\beta}(f) = \psi_{\alpha,\beta}(f) \right\}_{0 \leq \alpha \leq n, 0 \leq \beta \leq n}
\]

**Two SE with their own size parameter**

\[
\Pi^\psi(f) = \left\{ \Pi^\psi_{\alpha,\beta}(f) \mid \Pi^\psi_{\alpha,\beta}(f) = \psi_{b_{\alpha}}(f) + \psi_{c_{\beta}}(f) \right\}_{0 \leq \alpha \leq n, 0 \leq \beta \leq n}
\]

**Attribute filters**

\[
\Pi^{\psi_{\chi_1,\chi_2}}(f) = \left\{ \Pi^\psi_{\alpha,\beta}(f) \mid \Pi^\psi_{\alpha,\beta}(f) = \psi_{\chi_1}(f) \land \psi_{\chi_2}(f) \right\}_{0 \leq \alpha \leq n, 0 \leq \beta \leq n}
\]

where $\chi_1$ and $\chi_2$ are respectively related to the area and the ratio of the moment of inertia to the square of the area.
Size-shape

Illustration of the 2-D pattern spectrum

Pattern spectrum with square SE
Pattern spectrum with horizontal line SE
Pattern spectrum with vertical line SE

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Illustration of the 2-D pattern spectrum
Size-orientation

Filters with various orientations

$$\Pi^\psi(f) = \left\{ \Pi_{\lambda,\theta}^\psi(f) \mid \Pi_{\lambda,\theta}^\psi(f) = \psi_{\lambda,\theta}(f) \right\} \quad 0 \leq \lambda \leq n \quad \theta_0 \leq \theta \leq \theta_n$$

Other orientation-related features

- At each pixel $p$, keep the maximum/minimum value for each $\lambda$ (similar to radial filters)
- Compute orientation maps
Size-orientation

Illustration of the size-orientation granulometry

Granulometry with vertical line SE

SE size

Granulometry with vertical line SE

SE size
Size-orientation

Illustration of the size-orientation granulometry
Size-spectral

**Marginal strategy: the basic approach**

\[
\Pi^{\psi}(f) = \left\{ \Pi^{\psi}_{\lambda,\omega}(f) \mid \Pi^{\psi}_{\lambda,\omega}(f) = \psi_{\lambda}(f_{\omega}) \right\} \quad 1 \leq \omega \leq k
\[
0 \leq \lambda \leq n
\]

**Vectorial strategy: spectral correlation and vector preservation**

\[
\Pi^{\psi}(f) = \left\{ \Pi^{\psi}_{\lambda,\omega}(f) \mid \Pi^{\psi}_{\lambda,\omega}(f) = (\psi^v_{\lambda}(f))_{\omega} \right\} \quad 1 \leq \omega \leq k
\[
0 \leq \lambda \leq n
\]

with:

\[
\varepsilon^v_b(f)(p) = \inf_{q \in b} f(p + q), \quad p \in E
\]

\[
\delta^v_b(f)(p) = \sup_{q \in b} f(p - q), \quad p \in E
\]
Size-spectral granulometry

Illustration of the size-spectral granulometry
Illustration of the size-spectral granulometry
Size-intensity

Histogram from morphological scale-space

\[
\Pi^{\psi}(f) = \left\{ \Pi^{\psi}_{\lambda,\eta}(f) \mid \Pi^{\psi}_{\lambda,\eta}(f) = h^{\psi}_{\lambda}(\eta) \right\}_{\eta_0 \leq \eta \leq \eta_n}^{0 \leq \lambda \leq n}
\]

with \( h_f(\eta) = \sum_{p \in E} \delta_{\eta f(p)} \) or \( h'_f(\eta) = h_f(\eta) / \sum_{\eta' \in T} h_f(\eta') \)

With a structuring function \( g_{\lambda,\eta} \) defined as a cylinder of radius \( \lambda \) and amplitude \( \eta \)

\[
\Pi^{\psi}(f) = \left\{ \Pi^{\psi}_{\lambda,\eta}(f) \mid \Pi^{\psi}_{\lambda,\eta}(f) = \psi_{\lambda,\eta}(f) \right\}_{\eta_0 \leq \eta \leq \eta_n}^{0 \leq \lambda \leq n}
\]

Granolds with stacks of binary images

\[
\Pi^{\psi}(f) = \left\{ \Pi^{\psi}_{\lambda,\eta}(f) \mid \Pi^{\psi}_{\lambda,\eta}(f) = \psi_{\lambda}(T_{\eta}(f)) \right\}_{\eta_0 \leq \eta \leq \eta_n}^{0 \leq \lambda \leq n}
\]
Illustration of the size-intensity feature

- Grayscale histogram:
  - One bright large square and many dark small squares
  - Two dark large squares and many bright small squares

- Granulometric curve:
  - One bright large square and many dark small squares
  - Two dark large squares and many bright small squares
Illustration of the size-intensity feature

Size-intensity morphological feature

Size-intensity morphological feature
Introduction

Multiscale representation

1-D features

2-D features

Applications

Conclusion

Size-spatial

Spatial granulometry

\[ \Phi(f) = \{ \Phi_\lambda(f) \mid \Phi_\lambda(f) = m_{ij}(\Delta_\lambda(f)) \} \quad \text{with } m_{ij}(f) = \sum_{(x,y) \in E} x^i y^j f(x,y) \]

\[ \Phi_\lambda(f) = \{ \Phi_\lambda(f) = m_{ij}(\Delta_\lambda(f)) \} \quad \text{with } m_{ij}(f) = \sum_{(x,y) \in E} x^i y^j f(x,y) \]

Spatial covariance

\[ K^{\vec{\nu}}(f) = \left\{ K^{\vec{\nu}}_\lambda \mid K^{\vec{\nu}}_\lambda = \mu_{ij} \left( \Pi^\xi_{\lambda,\vec{\nu}}(f)(p) \right) / \mu_{ij}(f) \right\} \]

\[ \mu_{ij}(f) = \frac{\sum_{(x,y) \in E} (x-\bar{x})^i (y-\bar{y})^j f(x,y)}{(m_{00}(f))^\alpha} \quad \text{and } \alpha = \frac{i+j}{2} + 1, \quad \forall i + j \geq 2 \]

and \( \bar{x} = m_{10}(f) / m_{00}(f) \), \( \bar{y} = m_{01}(f) / m_{00}(f) \)
Size-spatial

Spatial size distributions

\[
\Omega(f) = \left\{ \begin{array}{l}
\Omega_{\lambda,\kappa} | \\
\Omega_{\lambda,\kappa} = \frac{1}{\left(\sum_{p \in E} f(p)\right)^2} \sum_{q \in \kappa} b K'_1(q)(f) - K'_1(\Pi_{\lambda}(f)) \end{array} \right\}_{0 \leq \kappa \leq k, 0 \leq \lambda \leq n}
\]

with \(\vec{q}\) a shortcut for \(\overrightarrow{oq}\), \(o\) the origin of the SE and \(K'\) the covariance built from autocorrelation functions, i.e.

\[
\varepsilon'_{\lambda,\vec{v}}(f)(p) = f(p - \lambda \vec{v}) \cdot f(p + \lambda \vec{v})
\]
Combining granulometry and covariance

\[ \Pi_{\psi, \vec{v}}(f) = \left\{ \Pi_{\lambda, \kappa}(f) \mid \Pi_{\lambda, \kappa}(f) = \psi_{\lambda, \kappa, \vec{v}}(f) \right\} \]

with \( \psi_{\lambda, \kappa, \vec{v}} \) a shortcut for \( \psi_{b_{\lambda, \kappa, \vec{v}}} \)

and the composite SE being defined as \( b_{\lambda, \kappa, \vec{v}} = b_{\lambda} \cup (b_{\lambda} + \kappa \vec{v}) \)

This feature can be normalized:

\[ \Gamma_{\psi, \vec{v}}(f) = \left\{ \Gamma_{\lambda, \kappa}(f) \mid \Gamma_{\lambda, \kappa}(f) = \frac{\sum_{p \in E} \Pi_{\lambda, \kappa}(f)(p)}{\sum_{p \in E} f(p)} \right\} \]
Size-spatial

Illustration of the spatial covariance

- Left: Illustration of the spatial covariance with a vertical line SE.
- Middle: Covariance with a vertical line SE.
- Right: Covariance with a vertical line SE.

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Size-spatial

Illustration of the spatial covariance

\[ \text{Spatial moment} \]

\[ \text{SE length} \]
Applications

Various problems

- texture classification
- texture segmentation
- shape recognition
- parameter settings
- image and video classification
- edge detection
- noise reduction
Various fields

- biomedical imaging
- remote sensing
- document analysis, signature recognition
- content-based image retrieval
- biometrics, silhouette recognition
Applications

Practical implementation issues

- efficient algorithms
  - fast structural filters
  - fast attribute filters
  - SE decomposition
  - dedicated hardware or parallel architecture

- robustness and adaptation
  - to noise
  - dimensionality reduction
  - data discretization
  - adaptation or learning
Conclusions and Perspectives

**Conclusion**

- MM is a relevant framework for scale-spaces
- Standard features (granulometry, pattern spectrum) may be insufficient
- 2-D features exist to gather complementary information

**Perspectives**

- Structural operators may be avoided
- Various efficient schemes exist to ensure real-time
- Other morphological scale-spaces:
  - Multiscale connectivites
  - PDE
  - Levelings or watersheds
Conclusions and Perspectives

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Thank you for your attention!

Any questions?