

DÉDUCTION NATURELLE

$$\frac{}{\Gamma \vdash A} \text{ si } A \in \Gamma \quad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B} \text{ Intro } \Rightarrow \quad \frac{\Gamma \vdash A \Rightarrow B \quad \Gamma \vdash A}{\Gamma \vdash B} \text{ Elim } \Rightarrow$$

Figure 1: Déduction naturelle: logique minimale

$$\frac{}{\Gamma \vdash A} \text{ si } A \in \Gamma \quad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B} \text{ Intro } \Rightarrow \quad \frac{\Gamma \vdash A \Rightarrow B \quad \Gamma \vdash A}{\Gamma \vdash B} \text{ Elim } \Rightarrow$$

$$\frac{\Gamma \vdash \perp}{\Gamma \vdash P} \text{ Elim } \perp$$

$$\frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A} \text{ Intro } \neg \quad \frac{\Gamma \vdash \neg A \quad \Gamma \vdash A}{\Gamma \vdash \perp} \text{ Elim } \neg$$

$$\frac{\Gamma \vdash P \quad \Gamma \vdash Q}{\Gamma \vdash P \wedge Q} \text{ Intro } \wedge \quad \frac{\Gamma \vdash P \wedge Q}{\Gamma \vdash P} \text{ Elim } \wedge g \quad \frac{\Gamma \vdash P \wedge Q}{\Gamma \vdash Q} \text{ Elim } \wedge d$$

$$\frac{\Gamma \vdash P}{\Gamma \vdash P \vee Q} \text{ Intro } \vee g \quad \frac{\Gamma \vdash Q}{\Gamma \vdash P \vee Q} \text{ Intro } \vee d$$

$$\frac{\Gamma \vdash P \vee Q \quad \Gamma, P \vdash R \quad \Gamma, Q \vdash R}{\Gamma \vdash R} \text{ Elim } \vee$$

Figure 2: Déduction naturelle: logique intuitionniste

$$\frac{\Gamma, \neg P \vdash \perp}{\Gamma \vdash P} \text{ RAA}$$

Figure 3: Déduction naturelle: logique classique

$$\frac{\Gamma \vdash A}{\Gamma \vdash \forall x A} \forall \text{ intro } (x \text{ n'est pas libre dans } \Gamma) \quad \frac{\Gamma \vdash \forall x A}{\Gamma \vdash A[x \leftarrow t]} \forall \text{ elim}$$

$$\frac{\Gamma \vdash A[x \leftarrow t]}{\Gamma \vdash \exists x A} \exists \text{ intro} \quad \frac{\Gamma \vdash \exists x A \quad \Gamma, A \vdash B}{\Gamma \vdash B} \exists \text{ elim } (x \text{ n'est pas libre dans } \Gamma \text{ ni } B)$$

Figure 4: Déduction naturelle: logique des prédicats