Sujet de stage de M2 Informatique

Portabilité des preuves formelles : traduction automatique de la bibliothèque Geocoq vers Lean

Please check out page 2 for an english version of this document.

Nicolas Magaud [1] (magaud@unistra.fr) et Julien Narboux [2] (narboux@unistra.fr)
Lab. ICube UMR 7357 CNRS Université de Strasbourg, France

Avec l'essor des assistants de preuve tels que Lean [7], Coq [4, 6] ou encore Isabelle/HOL [8], les développements formels sont de plus en plus nombreux et de plus en plus complexes. De plus, de mêmes concepts mathématiques et leurs propriétés peuvent être définis de différentes manières dans ces différents systèmes d'aide à la preuve.

La bibliothèque GeoCoq [3], développée en Coq, propose une formalisation des fondements de la géométrie, en s'appuyant sur différentes axiomatiques. On peut notamment citer Euclid, Hilbert et Tarski. Il s'agit d'une des plus grandes bibliothèques formelles Coq par sa taille. En s'appuyant sur l'infrastructure de Dedukti [2], cette bibliothèque a été portée avec succès vers Isabelle/HOL et Matita [1].

L'objectif de ce stage est d'étudier comment traduire la bibliothèque GeoCoq pour obtenir une bibliothèque équivalente dans l'assistant de preuves Lean. Il s'agira d'étudier comment relier les concepts de base des deux systèmes d'aide à la preuve, puis de traduire le plus automatiquement possible les lemmes et leurs preuves. Cela pourrait notamment passer par l'utilisation d'outils de type *hammer* comme c'est le cas dans une autre traduction récente [5] vers Isabelle/HOL. Les systèmes de preuve Coq et Lean ayant des fondements logiques très proches, on cherchera à faire une traduction directe, évitant ainsi un encodage de la logique de Coq dans Lean.

Références

[1] Andrea Asperti, Wilmer Ricciotti, Claudio Sacerdoti Coen, and Enrico Tassi. The Matita Interactive Theorem Prover. In Nikolaj Bjørner and Viorica Sofronie-Stokkermans, editors, *Automated Deduction – CADE-23*, pages 64–69, Berlin, Heidelberg, 2011. Springer Berlin Heidelberg.

[2] Ali Assaf, Guillaume Burel, Raphaël Cauderlier, David Delahaye, Gilles Dowek, Catherine Dubois, Frédéric Gilbert, Pierre Halmagrand, Olivier Hermant, and Ronan Saillard. Dedukti : a logical framework based on the $\lambda - \Pi$-calculus modulo theory. Manuscript `http://www.lsv.fr/~dowek/Publi/expressing.pdf`, 2016.

[3] Michael Beeson, Pierre Boutry, Gabriel Braun, Charly Gries, and Julien Narboux. GeoCoq : A formalization of foundations of geometry in Coq, June 2018.

[4] Yves Bertot and Pierre Castéran. *Interactive Theorem Proving and Program Development, Coq'Art : The Calculus of Inductive Constructions*. Texts in Theoretical Computer Science, An EATCS Series. Springer-Verlag, Berlin/Heidelberg, May 2004. 469 pages.

[5] Roland Coghetto and Julien Narboux. The GeoCoq library and its porting to Isabelle, 2022. EuroProofNet Workshop on the development, maintenance, refactoring and search of large libraries of proofs.

[6] Coq development team. *The Coq Proof Assistant Reference Manual, Version 8.14.0*, 2021.

[7] Leonardo Mendonça de Moura, Soonho Kong, Jeremy Avigad, Floris van Doorn, and Jakob von Raumer. The Lean Theorem Prover (System Description). In *Proceedings of CADE 2015*, volume 9195 of *LNCS*, pages 378–388. Springer, 2015.

[8] Tobias Nipkow, Lawrence C. Paulson, and Markus Wenzel. *Isabelle/HOL — A Proof Assistant for Higher-Order Logic*, volume 2283 of *LNCS*. Springer, 2002.

1. `https://dpt-info.u-strasbg.fr/~magaud/`
2. `https://dpt-info.u-strasbg.fr/~narboux/`

Master Internship in Computer Science

Interoperability of Proof Systems : Translating the GeoCoq Library into Lean

Nicolas Magaud[1] (magaud@unistra.fr) et Julien Narboux[2] (narboux@unistra.fr)
Lab. ICube UMR 7357 CNRS Université de Strasbourg, France

Proof assistants such as Lean [7], Coq [4, 6] or Isabelle/HOL [8] are increasingly popular and new proof developments are published everyday. These libraries get larger and more complex, relying on numerous mathematical concepts, usually implemented using very different styles in different proof assistants.

The GeoCoq library [3], developed in Coq, formalizes the foundations of geometry. It relies on the axiom systems proposed by Euclid, Hilbert and Tarski. It is one of the largest libraries of formal proofs implemented in Coq by its size. Using the proof interoperability infrastructure of Dedukti [2], this library has been successfully translated into two other proof assistants, namely Isabelle/HOL and Matita [1].

In this internship, we aim at translating automatically the GeoCoq library into an equivalent library for the Lean proof assistant. This would consist in linking the basic mathematical concepts of both proof systems and then translating, as automatically as possible, all the lemmas of the library as well as their proofs. We shall study whether proving the translated statements in the target proof assistant (Lean) could be achieved using *hammer*-like tools as it was the case in the recent translation [5] to Isabelle/HOL. Coq and Lean being two proof assistants with very close logical foundations, we suggest to carry out a direct translation, thus avoiding the step of encoding Coq logic into Lean.

References

[1] Andrea Asperti, Wilmer Ricciotti, Claudio Sacerdoti Coen, and Enrico Tassi. The Matita Interactive Theorem Prover. In Nikolaj Bjørner and Viorica Sofronie-Stokkermans, editors, *Automated Deduction – CADE-23*, pages 64–69, Berlin, Heidelberg, 2011. Springer Berlin Heidelberg.

[2] Ali Assaf, Guillaume Burel, Raphaël Cauderlier, David Delahaye, Gilles Dowek, Catherine Dubois, Frédéric Gilbert, Pierre Halmagrand, Olivier Hermant, and Ronan Saillard. Dedukti: a logical framework based on the $\lambda - \Pi$-calculus modulo theory. Manuscript `http://www.lsv.fr/~dowek/Publi/expressing.pdf`, 2016.

[3] Michael Beeson, Pierre Boutry, Gabriel Braun, Charly Gries, and Julien Narboux. GeoCoq: A formalization of foundations of geometry in Coq, June 2018.

[4] Yves Bertot and Pierre Castéran. *Interactive Theorem Proving and Program Development, Coq'Art : The Calculus of Inductive Constructions*. Texts in Theoretical Computer Science, An EATCS Series. Springer-Verlag, Berlin/Heidelberg, May 2004. 469 pages.

[5] Roland Coghetto and Julien Narboux. The GeoCoq library and its porting to Isabelle, 2022. EuroProofNet Workshop on the development, maintenance, refactoring and search of large libraries of proofs.

[6] Coq development team. *The Coq Proof Assistant Reference Manual, Version 8.14.0*, 2021.

[7] Leonardo Mendonça de Moura, Soonho Kong, Jeremy Avigad, Floris van Doorn, and Jakob von Raumer. The Lean Theorem Prover (System Description). In *Proceedings of CADE 2015*, volume 9195 of *LNCS*, pages 378–388. Springer, 2015.

[8] Tobias Nipkow, Lawrence C. Paulson, and Markus Wenzel. *Isabelle/HOL — A Proof Assistant for Higher-Order Logic*, volume 2283 of *LNCS*. Springer, 2002.

[1] `https://dpt-info.u-strasbg.fr/~magaud/`
[2] `https://dpt-info.u-strasbg.fr/~narboux/`