# A code in the protein coding genes 

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#### Abstract

A statistical analysis with 12288 autocorrelation functions applied in protein (coding) genes of prokaryotes and eukaryotes identifies three subsets of trinucleotides in their three frames: $T_{0}=X_{0} \cup\{A A A, T T T\}$ with $X_{0}=\{A A C$, AAT, ACC, ATC, ATT, CAG, CTC, CTG, GAA, GAC, GAG, GAT, GCC, GGC, GGT, GTA, GTC, GTT, TAC, TTC\} in frame 0 (the reading frame established by the ATG start trinucleotide), $\mathrm{T}_{1}=\mathrm{X}_{1} \cup\{C C C\}$ in frame 1 and $T_{2}=X_{2} \cup\{G G G\}$ in frame 2 (the frames 1 and 2 being the frame 0 shifted by one and two nucleotides, respectively, to the right). These three subsets are identical in these two gene populations and have five important properties: (i) the property of maximal ( 20 trinucleotides) circular code for $X_{0}\left(\right.$ resp. $X_{1}, X_{2}$ ) allowing to retrieve automatically the frame 0 (resp. 1, 2) in any region of the gene without start codon; (ii) the DNA complementarity property $\mathscr{C}$ (e.g. $\mathscr{C}(\mathrm{AAC})=\mathrm{GTT}): \mathscr{C}\left(\mathrm{T}_{0}\right)=\mathrm{T}_{0}, \mathscr{C}\left(\mathrm{~T}_{1}\right)=\mathrm{T}_{2}$ and $\mathscr{C}\left(\mathrm{T}_{2}\right)=\mathrm{T}_{1}$ allowing the two paired reading frames of a DNA double helix simultaneously to code for amino acids; (iii) the circular permutation property $\mathscr{P}$ (e.g. $\mathscr{P}(\mathrm{AAC})=\mathrm{ACA}$ ): $\mathscr{P}\left(\mathrm{X}_{0}\right)=\mathrm{X}_{1}$ and $\mathscr{P}\left(\mathrm{X}_{1}\right)=\mathrm{X}_{2}$ implying that the two subsets $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$ can be deduced from $\mathrm{X}_{0}$; (iv) the rarity property with an occurrence probability of $X_{0}=6 \times 10^{-8}$; and (v) the concatenation properties in favour of an evolutionary code: a high frequency ( $27.5 \%$ ) of misplaced trinucleotides in the shifted frames, a maximum ( 13 nucleotides) length of the minimal window to retrieve automatically the frame and an occurrence of the four types of nucleotides in the three trinucleotide sites. In Discussion, a simulation based on an independent mixing of the trinucleotides of $T_{0}$ allows to retrieve the two subsets $T_{1}$ and $T_{2}$. Then, the identified subsets $T_{0}, T_{1}$ and $T_{2}$ replaced in the 2-letter genetic alphabet $\{R, Y\}(R=$ purine $=A$ or $G, Y=$ pyrimidine $=C$ or $T)$ allow to retrieve the $R N Y$ model $(N=R$ or $Y)$ and to explain previous works in the alphabet $\{R, Y\}$. Then, these three subsets are related to the genetic code. The trinucleotides of $\mathrm{T}_{0}$ code for 13 amino acids: Ala, A sn, Asp, GIn, Glu, Gly, Ile, Leu, Lys, Phe, Thr, Tyr and Val. Finally, a strong correlation between the usage of the trinucleotides of $\mathrm{T}_{0}$ in protein genes and the amino acid frequencies in proteins is observed as six among seven amino acids not coded by $\mathrm{T}_{0}$, have as expected the lowest frequencies in proteins of both prokaryotes and eukaryotes. © 1997 Elsevier Science Ireland Ltd.


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## 1. Introduction

The concept of code without comma introduced by Crick et al. (1957) is a code readable in only one frame and without a start signal. Such a theoretical code 'without comma' is a set $X$ of codons so that their concatenation (series of codons) leads to genes which have the interesting property to retrieve automatically the concatenation of codons of $X$ without the usage of a start codon in the case of the trace of this initial concatenation is lost (the 'commas' dividing the series of nucleotides into groups of three for constituting the codons in the initial concatenation). Such a code was proposed in order to explain how the reading of a series of nucleotides in the protein (coding) genes could code for the amino acids constituting the proteins. The two problems stressed were: why are there more codons than amino acids and how to choose the reading frame? For example, a series of nucleotides ...AGTCCGTACGA... can be read in three frames: ...AGT, CCG, TAC, GA..., ...A, GTC, CGT, ACG, A... and ...AG, TCC, GTA, CGA, ... Crick et al. (1957) have then proposed that only 20 among 64 codons, code for the 20 amino acids. However, the determination of a set of 20 codons forming a code $X$ without commas depends on a great number of constraints. For example, the four codons with identical nucleotides AAA, CCC, GGG and TTT must be excluded from such a code. Indeed, the concatenation of AAA, for example, with itself does not allow to retrieve the frame: ...AAA, $A A A, A A A, \ldots, \ldots A, A A A, A A A, A A \ldots$ and ... AA, AAA, AAA, A... Similarly, two codons related to circular permutation, e.g. AAC and ACA (or CAA), cannot belong at the same time to such a code. Indeed, the concatenation of $A A C$, for example, with itself leads to the concatenation of ACA (or CAA) with itself in another frame, making the frame determination impossible. Therefore, by excluding AAA, CCC, GGG and TTT and by gathering the 60 re-
maining codons in 20 classes of three codons so that, in each class, the three codons are deduced from each other by circular permutations, e.g. AAC, ACA and CAA, a code without commas has only one codon per class and therefore contains at most 20 codons. This codon number is identical to the amino acid number. This remark has naturally led to propose a code without commas assigning one codon per amino acid (Crick et al., 1957).
In contrast, D ounce (1952) has earlier proposed an evolutionary code associating several codons per amino acid. Such a flexibility can explain the variations in $G+C$ composition observed in the actual protein genes (Jukes and Bhushan, 1986).
The two discoveries that the codon TTT, an 'excluded' codon in the concept of code without commas, codes for phenylalanine ( N irenberg and M atthaei, 1961) and that the protein genes are placed in the reading frame with a particular codon, namely the start codon ATG, have led to give up the concept of code without commas in the alphabet $\{A, C, G, T\}$. F or several biological reasons, in particular the interaction between mRNA and tRNA, the concept of code without commas is resumed later in the alphabet $\{\mathrm{R}, \mathrm{Y}\}$ ( $\mathrm{R}=$ purine $=\mathrm{A}$ or $\mathrm{G}, \mathrm{Y}=$ pyrimidine $=\mathrm{C}$ or T ) with two codon models for the primitive protein genes: R R Y (Crick et al., 1976) and R N Y ( $\mathrm{N}=\mathrm{R}$ or Y) (Eigen and Schuster, 1978).

In order to understand the circular code identified here in protein genes of prokaryotes and eukaryotes in the alphabet $\{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$, the concept of circular code is introduced in the alphabet $\{R, Y\}$ with the $R N Y$ codon model (Eigen and Schuster, 1978). If a sequence, e.g. a protein gene, is constructed by concatenating trinucleotides of the type RNY, i.e. RRY and RYY forming the frame 0 (reading frame), and if the frame of construction is lost, e.g. situation observed when a region of a protein gene without a start codon is sequenced, then the property of code assures that the constructed sequence can be decomposed into a series of RNY trinucleotides
according to a unique way. This unique decomposition, and therefore the frame 0 , can be retrieved using a window of nucleotides with a minimal length depending on RNY. In the actual protein genes, the frame 0 is determined by the start codon. The notion of circular added to the concept of code concerns the limit case with sequences of infinite length, i.e. without a beginning and an end.

RNY is a circular code. Indeed, the concatenation of two trinucleotides of RNY, ...RRYRRY ..., ...RRYRYY ..., ...RYYRRY ... and ... RY Y R Y Y ... leads to only one decomposition over R NY. This because the eventual decomposition in frame 1 has always an $R$ in the 3rd position but no trinucleotide of RNY ends with R and as the eventual decomposition in frame 2 has always an $Y$ in the first position but no trinucleotide of RNY begins with Y. The RNY codon model leads to a protein gene formed by a series RNYRNY... of nucleotides so that there is one type of trinucleotide RNY in frame 0 (reading frame), one type of trinucleotide N Y R in frame 1 and one type of trinucleotide YRN in frame 2 (frames 1 and 2 being the frame 0 shifted by one and two nucleotides, respectively, in the $5^{\prime}-3^{\prime}$ direction). RNY is self-complementary and, NYR and YRN are complementary to each other. This property allows the two paired reading frames to simultaneously code for amino acids according to a purine/pyrimidine genetic code. Furthermore, NYR (resp. YRN) is obtained by one (resp. 2) circular permutation of RNY. This property allows $N Y R$ and $Y R N$ to be deduced from RNY. Finally, the length of the minimal window to automatically retrieve the frame 0 in a series RNYRNY... is obviously equal to three nucleotides. Indeed, two nucleotides are insufficient as RY is both in frame 1 of RRY (RNY with $N=R$ ) and in frame 0 of RYY ( $R N Y$ with $\mathrm{N}=\mathrm{Y}$ ). This property allows to automatically retrieve the reading frame in any region of the gene (formed by a series of R NY codons) without a start codon.

Recently, the 64 autocorrelation functions analyzing in the protein genes of prokaryotes and eukaryotes, the occurrence probability of two trinucleotides obtained by specifying Y RY on $\{A$,

C, G, T \} (YRY leads to eight specified trinucleotides \{CAC, CAT, ..., TGT\}) and separated by any i bases $\mathrm{N}(\mathrm{N}=\mathrm{A}$ or C or G or T$)$, have allowed to retrieve the classical periodicity 0 modulo 3 (maximal values of the function at $\mathrm{i}=0,3$, 6 , etc.) and unexpectedly, to identify shifted modulo 3 periodicities: 1 modulo 3 (maximal values of the function at $i=1,4,7$, etc.) and 2 modulo 3 (maximal values of the function at $\mathrm{i}=2,5,8$, etc.) (A rquès et al., 1995). This result means that these eight trinucleotides are associated with the three frames of protein genes, some trinucleotides with the reading frame, and others with the reading frame shifted by one or two nucleotides in the $5^{\prime}-3^{\prime}$ direction. However, as the definition used is based on the average of the three frames 0,1 and 2, it is impossible to associate a particular frame with a given trinucleotide.

In order to extend this result to the 64 trinucleotides $\{$ AAA,$\ldots$, TTT $\}$ and to identify subsets of trinucleotides having a preferential occurrence frame, the previous approach is generalized here to the 12288 autocorrelation functions analyzing the probability that a trinucleotide in any frame occurs any i bases N after a trinucleotide in a given frame of the protein genes. This statistical analysis identifies three subsets of trinucleotides per frame in protein genes of both prokaryotes and eukaryotes which have five important properties: maximal circular code, DN A complementarity, circular permutation, rarity and concatenation. Therefore, the code identified here in protein genes of prokaryotes and eukaryotes in the alphabet $\{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$ retrieves the properties both of the code RNY in the reduced alphabet $\{R, Y\}$ (Eigen and Schuster, 1978) and of the evolutionary code (D ounce, 1952).

## 2. M ethod

2.1. Definition of the autocorrelation function in
frame frame

This definition generalizes the autocorrelation function definition in the alphabet $\{\mathrm{R}, \mathrm{Y}\}$ and without considering the frame (A rquès and Michel, 1987) to the alphabet $\mathscr{B}=\{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$
and in frame. For a gene population $F$, two trinucleotides $w$ and $w$ ' and a frame $p$, the 'autocorrelation function in frame' noted $w^{p}(N)_{i} w^{\prime}$ is defined as the function of $i$ giving the occurrence probability in the gene population $F$ that the trinucleotide $\mathrm{w}^{\prime}$ in any frame occurs any i bases N after the trinucleotide $w$ read in frame $p$.
$M$ ore precisely, let $F$ be a gene population (language) with $n(F)$ protein (coding) genes (words). Let s be a protein gene in F of length $\mathrm{I}(\mathrm{s})$. Let w be a trinucleotide (word of three letters) on $\mathscr{B}, \mathrm{w} \in \mathrm{T}=\{\mathrm{AAA}, \ldots, \mathrm{TTT}\}$ ( 64 trinucleotides). A trinucleotide $w$ read in frame $p$ in a protein gene is noted $w^{p}, p \in\{0,1,2\}(64 \times 3=192$ trinucleotides in the three frames). Let the i-motif $\mathrm{m}_{\mathrm{i}}^{\mathrm{p}}=$ $w^{p}(N)_{i} w^{\prime}$ be two trinucleotides $w^{p}$ in frame $p$ and $w^{\prime}$ in any frame separated by any $i$ bases $N$, with $w, w^{\prime} \in T, p \in\{0,1,2\}, i \in[0,99]$ and $N=A, C, G$ or $\quad T$, i.e. $\quad m_{i}^{p} \in\left\{A A A^{p}(N)_{i} A A A, \quad \ldots\right.$, $\operatorname{TTT}^{\mathrm{p}}(\mathrm{N})_{\mathrm{i}}$ TTT\} $\quad\left(64^{2} \times 3=12288\right.$ i-motifs $)$. For each protein gene $s$ of $F$, the counter $c_{p}^{p}(s)$ counts the occurrences of $\mathrm{m}_{\mathrm{p}}^{\mathrm{p}}$ in s . The occurrence probability $\mathrm{o}^{\mathrm{p}}(\mathrm{s})$ of $\mathrm{m}_{\mathrm{p}}^{\mathrm{p}}$ in s , is then equal to the ratio of the counter by the total number $t(s)$ of trinucleotides read in frame $p, o p(s)=c_{p}^{p}(s) / t(s)$ with $\mathrm{t}(\mathrm{s})=(\mathrm{l}(\mathrm{s})-104) / 3 \quad(104=99+2 \times 3-1) . \quad$ The occurrence probability $A_{w, w}^{p}(i, F)$ of the i-motif $m_{i}^{p}$ in a gene population $F$, is $A_{w, w^{\prime}}^{p}(i, F)=[1 /$ $\mathrm{n}(\mathrm{F})] \times \Sigma_{\mathrm{s} \in \mathrm{F}} \quad \mathrm{op}$.

For a gene population $F$, the autocorrelation function $W^{p}(N)_{i} W^{\prime}$ is then the function $i \rightarrow$ $\mathrm{A}_{\mathrm{w}, \mathrm{w}}^{\mathrm{p}}(\mathrm{i}, \mathrm{F})$ giving the occurrence probability that $\mathrm{w}^{\prime}$ in any frame occurs any i bases N after $\mathrm{w}^{\mathrm{p}}$ in frame $p$ and is represented as a curve as follows: (i) the abscissa shows the number i of bases N between $w^{\mathrm{p}}$ and $\mathrm{w}^{\prime}$, by varying i between 0 and 99 (for clarity reasons, figures are given with a lesser number of points); (ii) the ordinate gives the occurrence probability of $w^{\mathrm{P}}(\mathrm{N})_{\mathrm{i}} \mathrm{w}^{\prime}$ in a gene population $F$.

### 2.2. Data acquisition

The two gene populations $F$ analyzed here with the 12288 autocorrelation functions are the protein genes of prokaryotes $F=$ CPRO (13 686 sequences, 14167 kb ) and (nuclear) eukaryotes $\mathrm{F}=\mathrm{CEUK}$ (26757 sequences, 34227 kb ). The
sequences with a minimal length of 200 bases are obtained from the release 39 of the EM BL N ucleotide Sequence Data Library in the same way as described in previous papers (A rquès and $M$ ichel, 1990a,b for a description of data acquisition). These large populations allow to obtain significant statistical results.

## 3. Results

### 3.1. Identification of shifted periodicities in protein genes

The 12288 autocorrelation functions applied in the protein genes of prokaryotes and eukaryotes are non-random. Indeed, almost all autocorrelation functions have a modulo 3 periodicity among the three following types:
(i) Type 0 : a periodicity 0 modulo 3 (maximal values of the function at $i=0,3,6$, etc.), three examples are given in Fig. 1(a)-(c) with $\mathrm{F}=$ CPRO;
(ii) Type 1: a periodicity 1 modulo 3 (maximal values of the function at $i=1,4,7$, etc.), three examples are given in Fig. 1(d)-(f) with $\mathrm{F}=$ CPRO;
(iii) Type 2: a periodicity 2 modulo 3 (maximal values of the function at $\mathrm{i}=2,5,8$, etc.), three examples are given in Fig. 1(g)-(i) with $\mathrm{F}=$ CPRO.

The identification of shifted periodicities (types 1 and 2) in protein genes is new.

The method to classify the autocorrelation function $w^{\mathrm{p}}(\mathrm{N}) \mathrm{w}^{\mathrm{w}}, \mathrm{w}, \mathrm{w}^{\prime}$ in T and p in $\{0,1,2\}$, consists in determining the number of points of each modulo 3 type which are greater than their two adjacent points. The maximal number is 33 points for a curve with 100 points ( $i \in[0,99]$ ). If the number of points for the type $p$ is $>22$ (corresponding to a statistical level of significance $<10^{-4}$ ), then the autocorrelation function is said to be classifiable according to this type $p$. With the statistical level chosen, an autocorrelation function can only be classifiable in one type. A few autocorrelation functions cannot be classifiable, in particular an autocorrelation function with an i-motif containing a stop trinucleotide is


Fig. 1. (a) Periodicity 0 modulo 3 in frame 0 of the prokaryotic protein coding genes. The horizontal axis represents the number i , $\mathrm{i} \in[0,30]$, of any bases N between $\mathrm{CGC}^{0}$ in frame 0 and GTC in any frame. The vertical axis represents the autocorrelation function $\mathrm{CGC}^{\circ}(\mathrm{N})_{\mathrm{i}} \mathrm{GTC}$; (b) periodicity 0 modulo 3 in frame 1 of the prokaryotic protein coding genes. The horizontal axis represents the number $i, i \in[0,30]$, of any bases $N$ between ATC ${ }^{1}$ in frame 1 and TCG in any frame. The vertical axis represents the autocorrelation function $\mathrm{ATC}^{1}(\mathrm{~N})_{\mathrm{i}} \mathrm{TCG}$; (c) periodicity 0 modulo 3 in frame 2 of the prokaryotic protein coding genes. The horizontal axis represents the number $\mathrm{i}, \mathrm{i} \in[0,30]$, of any bases N between CCC $^{2}$ in frame 2 and CGT in any frame. The vertical axis represents the autocorrelation function $\mathrm{CCC}^{2}(\mathrm{~N})_{\mathrm{i}} \mathrm{CGT}$; (d) periodicity 1 modulo 3 in frame 0 of the prokaryotic protein coding genes. The horizontal axis represents the number $\mathrm{i}, \mathrm{i} \in[0,30]$, of any bases N between $\mathrm{ATC}^{0}$ in frame 0 and TCG in any frame. The vertical axis represents the autocorrelation function $\mathrm{ATC}^{\circ}(\mathrm{N})_{\mathrm{i}} \mathrm{TCG}$; (e) periodicity 1 modulo 3 in frame 1 of the prokaryotic protein coding genes. The horizontal axis represents the number $i, i \in[0,30]$, of any bases $N$ between CCC $^{1}$ in frame 1 and CGT in any frame. The vertical axis represents the autocorrelation function $\mathrm{CCC}^{1}(\mathrm{~N})_{\mathrm{i}} \mathrm{CGT}$; (f) periodicity 1 modulo 3 in frame 2 of the prokaryotic protein coding genes. The horizontal axis represents the number $\mathrm{i}, \mathrm{i} \in[0,30]$, of any bases N between CGC${ }^{2}$ in frame 2 and GTC in any frame. The vertical axis represents the autocorrelation function $\mathrm{CGC}^{2}(\mathrm{~N})_{\mathrm{i}} \mathrm{GTC}$; $(\mathrm{g})$ periodicity 2 modulo 3 in frame 0 of the prokaryotic protein coding genes. The horizontal axis represents the number $i, i \in[0,30]$, of any bases $N$ between CCC ${ }^{0}$ in frame 0 and CGT in any frame. The vertical axis represents the autocorrelation function $\mathrm{CCC}^{\circ}(\mathrm{N})_{\mathrm{i}} \mathrm{CGT}$; (h) periodicity 2 modulo 3 in frame 1 of the prokaryotic protein coding genes. The horizontal axis represents the number $i, i \in[0,30]$, of any bases $N$ between $\mathrm{CG}^{1}$ in frame 1 and GTC in any frame. The vertical axis represents the autocorrelation function $\mathrm{CGC}^{1}(\mathrm{~N})_{\mathrm{i}} \mathrm{GTC}$; (i) periodicity 2 modulo 3 in frame 2 of the prokaryotic protein coding genes. The horizontal axis represents the number $i, i \in[0,30]$, of any bases N between $A T C^{2}$ in frame 2 and TCG in any frame. The vertical axis represents the autocorrelation function $A T C^{2}(N)_{\mathrm{i}} \mathrm{T} C \mathrm{G}$.


Fig. 1. (Continued)
equal to 0 (TAA, TAG or TGA do not occur in frame 0).
These three types of periodicities exist whatever the frame $p$ of the trinucleotide $w^{p}$. With the examples given previously, the periodicity 0 modulo 3 can be observed in frame 0 (e.g. the autocorrelation function $\mathrm{CGC}^{\circ}(\mathrm{N})_{\mathrm{i}} \mathrm{GTC}$ : Fig. 1(a)), in frame 1 (e.g. the autocorrelation function $\mathrm{ATC}^{1}(\mathrm{~N})_{\mathrm{i}} \mathrm{TCG}: \mathrm{Fig} .1(\mathrm{~b})$ ) and in frame 2 (e.g. the autocorrelation function $\operatorname{CCC}^{2}(\mathrm{~N})_{\mathrm{i}} \mathrm{CGT}$ : Fig. 1(c)). Similarly, the periodicity 1 (resp. 2) modulo 3 can be observed in frame 0 (e.g. resp. the autocorrelation functions $\mathrm{ATC}^{\circ}(\mathrm{N})_{\mathrm{i}} \mathrm{TCG}$ : Fig.
$1(d)$ and $\operatorname{CCC}^{\circ}(\mathrm{N})_{\mathrm{i}} \mathrm{CGT}$ : Fig. $1(\mathrm{~g})$ ), in frame 1 (e.g. resp. the autocorrelation functions $\mathrm{CCC}^{1}(\mathrm{~N})_{\mathrm{i}} \mathrm{CGT}$ : Fig. $1(\mathrm{e})$ and $\mathrm{CGC}^{1}(\mathrm{~N})_{\mathrm{i}} \mathrm{GTC}$ : Fig. $1(\mathrm{~h})$ ) and in frame 2 (e.g. resp. the autocorrelation functions $\mathrm{CGC}^{2}(\mathrm{~N})_{\mathrm{i}} \mathrm{GTC}$ : Fig. $1(\mathrm{f})$ and $\operatorname{ATC}^{2}(N)_{\mathrm{i}} \mathrm{TCG}:$ Fig. 1(i)).

If the autocorrelation function $\mathrm{w}^{\mathrm{p}}(\mathrm{N})_{\mathrm{i}} \mathrm{w}^{\prime}, \mathrm{w}, \mathrm{w}^{\prime}$ in $T$ and $p$ in $\{0,1,2\}$, has a periodicity $j$ modulo 3 then the trinucleotide $w^{\prime}$ preferentially occurs in frame $q(w, p)=p+j$ modulo 3 after $w$ in frame $p$. Table 1 illustrates this remark with the previous examples (Fig. 1(a)-(i)). Table 1 also serves to introduce the next section by showing with these

Table 1
Frame $q=p+j$ of $w^{\prime}$ deduced from the frame $p$ of $w$ and the type of modulo 3 periodicity identified with the autocorrelation functions $w^{p}(N)_{i} w^{\prime}$ given in Fig. 1(a)- (i)

| A utocorrelation function $W^{p}(N)_{i} w^{\prime}$ | $\begin{aligned} & \text { Periodicity } j=0 \\ & \text { modulo } 3 \end{aligned}$ | $\text { Periodicity } \mathrm{j}=1$ $\text { modulo } 3$ | Periodicity j=2 $\text { modulo } 3$ | $\mathrm{w}^{\prime}$ in frame $\mathrm{q}=\mathrm{p}+\mathrm{j}$ modulo 3 |
| :---: | :---: | :---: | :---: | :---: |
| $A T C^{p}=0(N) i_{i}$ TCG |  | Fig. 1(d) |  | TCG in frame $\mathrm{p}+\mathrm{j}=0+1=1$ modulo 3 |
| $A T C^{p=1}(N)_{\mathrm{i}} \mathrm{TCG}^{\text {a }}$ | Fig. 1(b) |  |  | TCG in frame $p+j=1+0=1$ modulo 3 |
| $\left.A T C{ }^{p}=2(N)\right)_{i}$ TCG |  |  | Fig. 1(i) | TCG in frame $\mathrm{p}+\mathrm{j}=2+2=1$ modulo 3 |
| $\mathrm{CCC}^{p=0}(\mathrm{~N})_{\mathrm{i}} \mathrm{CGT}$ |  |  | Fig. 1(g) | CGT in frame $\mathrm{p}+\mathrm{j}=0+2=2$ modulo 3 |
| $\mathrm{CCC}^{\mathrm{p}}=1(\mathrm{~N})_{\mathrm{i}} \mathrm{CGT}$ |  | Fig. 1(e) |  | CGT in frame $\mathrm{p}+\mathrm{j}=1+1=2$ modulo 3 |
| $\mathrm{CCC}^{\mathrm{p}}=2(\mathrm{~N})_{\mathrm{i}} \mathrm{CGT}$ | Fig. 1(c) |  |  | CGT in frame $\mathrm{p}+\mathrm{j}=2+0=2$ modulo 3 |
| CGC ${ }^{p=0}(\mathrm{~N})_{\mathrm{i}} \mathrm{GTC}$ | Fig. 1(a) |  |  | GTC in frame $\mathrm{p}+\mathrm{j}=0+0=0$ modulo 3 |
| CGC ${ }^{p}=1(\mathrm{~N})_{\mathrm{i}} \mathrm{GTC}$ |  |  | Fig. 1(h) | GTC in frame $\mathrm{p}+\mathrm{j}=1+2=0$ modulo 3 |
| $\mathrm{CGC}^{p}=2(\mathrm{~N})_{\mathrm{i}} \mathrm{GTC}$ |  | Fig. 1(f) |  | GTC in frame $\mathrm{p}+\mathrm{j}=2+1=0$ modulo 3 |

examples that the three trinucleotides $\mathrm{w}^{\prime}=\mathrm{CGT}$, GTC and TCG have a constant preferential occurrence frame 2, 0 and 1 , respectively, independent both of the three trinucleotides $w=A T C$, CCC and CGC and their frame $p$.

### 3.2. Preferential occurrence frame of a trinucleotide

Property 1: for each trinucleotide w' in T (with a few exceptions given below), the autocorrelation functions $\mathrm{w}^{\mathrm{p}}(\mathrm{N})_{\mathrm{i}} \mathrm{w}^{\prime}$ obtained by varying w in T and $p$ in $\{0,1,2\}$, have a periodicity j modulo 3 implying a constant preferential occurrence frame $q(w, p)$ in $\{0,1,2\}(=p+j$ modulo 3$)$ for $w^{\prime}$ independent of $w$ and $p$.
For a trinucleotide $w^{\prime}$, the classifiable autocorrelation function $w^{p}(N)_{i} w^{\prime}$ implies a frame $q$ for $w^{\prime}$. A maximum of $64 \times 3=192$ autocorrelation functions are classifiable. As the three stop trinucleotides do not occur in frame 0, only $192-3=$ 189 autocorrelation functions are classifiable in protein genes. For each trinucleotide $w^{\prime}$ and for each frame $q$ in the protein genes of prokaryotes (resp. eucaryotes), Table 2(a) (resp. Table 2(b))
gives the number of classifiable autocorrelation functions $\mathrm{w}^{\mathrm{p}}(\mathrm{N})_{\mathrm{i}} \mathrm{w}^{\prime}$ implying the frame q for $\mathrm{w}^{\prime}$. Very unexpectedly, these two Table 2(a,b) show that the trinucleotides w' have a constant preferential occurrence frame and can easily be classified in three subsets of trinucleotides according to the frame (Table 3(a)). The 22 trinucleotides in frame 0 form the subset $T_{0}=\{A A A, A A C, A A T, A C C$, ATC, ATT, CAG, CTC, CTG, GAA, GAC, GAG, GAT, GCC, GGC, GGT, GTA, GTC, GTT, TAC, TTC, TTT \} and the 21 trinucleotides in each of the frames 1 and 2 , the subsets $T_{1}$ and $T_{2}$, respectively, $T=T_{0} \cup T_{1} \cup T_{2}$ with $T_{1}$ and $T_{2}$ defined in Table 3(a). By considering the four trinucleotides with identical nucleotides, three subsets $X_{0}, X_{1}$ and $X_{2}$ of 20 trinucleotides can be defined from $T_{0}, T_{1}$ and $T_{2}: X_{0}=T_{0}-\{A A A$, TTT $\}, X_{1}=T_{1}-\{C C C\}$ and $X_{2}=T_{2}-\{G G G\}$. The same three subsets $T_{0}, T_{1}$ and $T_{2}$ are retrieved for the two populations. The few trinucleotides, classified into two frames or misclassified (two for the prokaryotes and five for the eukaryotes), have been assigned to the frame according to the properties identified with the other trinucleotides, both in prokaryotes and eukaryotes.

Table 2
For each trinucleotide $w^{\prime}$ and for each frame $q$ in the prokaryotic protein coding genes (a) and in the eukaryotic protein coding genes (b) the number of classifiable autocorrelation functions $w^{p}(N)_{i} w^{\prime}$ implying the frame $q$ for $w^{\prime}$ is given

| $\mathrm{w}^{\prime}$ in $\mathrm{q}=0$ | N umber | $\mathrm{w}^{\prime}$ in $\mathrm{q}=1$ | N umber | $w^{\prime}$ in $\mathrm{q}=2$ | N umber | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (a) Prokaryotic protein coding genes |  |  |  |  |  |  |
| AAA | 189 | AAA | 0 | AAA | 0 | 189 |
| AAC | 161 | AAC | 3 | AAC | 5 | 169 |
| AAG | 2 | AAG | 187 | AAG | 0 | 189 |
| AAT | 189 | AAT | 0 | AAT | 0 | 189 |
| ACA | 19 | ACA | 161 | ACA | 0 | 180 |
| ACC | 188 | ACC | 0 | ACC | 0 | 188 |
| ACG | 0 | ACG | 189 | ACG | 0 | 189 |
| ACT | 75 | ACT | 79 | ACT | 17 | 171 |
| AGA | 0 | AGA | 29 | AGA | 154 | 183 |
| AGC | 0 | AGC | 99 | AGC | 87 | 186 |
| AGG | 0 | AGG | 127 | AGG | 54 | 181 |
| AGT | 0 | AGT | 53 | AGT | 129 | 182 |
| ATA | 0 | ATA | 189 | ATA | 0 | 189 |
| ATC | 185 | ATC | 1 | ATC | 0 | 186 |
| ATG | 17 | ATG | 163 | ATG | 0 | 180 |
| ATT | 189 | ATT | 0 | ATT | 0 | 189 |
| CAA | 7 | CAA | 4 | CAA | 165 | 176 |
| CAC | 0 | CAC | 4 | CAC | 183 | 187 |
| CAG | 110 | CAG | 78 | CAG | 0 | 188 |
| CAT | 0 | CAT | 0 | CAT | 189 | 189 |
| CCA | 5 | CCA | 179 | CCA | 0 | 184 |
| CCC | 0 | CCC | 146 | CCC | 20 | 166 |
| CCG | 0 | CCG | 189 | CCG | 0 | 189 |
| CCT | 22 | CCT | 0 | CCT | 155 | 177 |
| CGA | 0 | CGA | 0 | CGA | 189 | 189 |
| CGC | 0 | CGC | 0 | CGC | 189 | 189 |
| CGG | 0 | CGG | 0 | CGG | 188 | 188 |
| CGT | 4 | CGT | 0 | CGT | 178 | 182 |
| CTA | 0 | CTA | 97 | CTA | 92 | 189 |
| CTC | 123 | CTC | 53 | CTC | 0 | 176 |
| CTG | 141 | CTG | 42 | CTG | 0 | 183 |
| CTT | 30 | CTT | 0 | CTT | 136 | 166 |
| GAA | 189 | GAA | 0 | GAA | 0 | 189 |
| GAC | 189 | GAC | 0 | GAC | 0 | 189 |
| GAG | 189 | GAG | 0 | GAG | 0 | 189 |
| GAT | 149 | GAT | 0 | GAT | 31 | 180 |
| GCA | 94 | GCA | 37 | GCA | 39 | 170 |
| GCC | 189 | GCC | 0 | GCC | 0 | 189 |
| GCG | 59 | GCG | 81 | GCG | 0 | 140 |
| GCT | 74 | GCT | 0 | GCT | 108 | 182 |
| GGA | 4 | GGA | 0 | GGA | 183 | 187 |
| GGC | 184 | GGC | 0 | GGC | 4 | 188 |
| GGG | 0 | GGG | 0 | G G G | 186 | 186 |
| GGT | 98 | GGT | 0 | GGT | 88 | 186 |
| GTA | 70 | GTA | 82 | GTA | 0 | 152 |
| GTC | 189 | GTC | 0 | GTC | 0 | 189 |
| GTG | 110 | GTG | 72 | GTG | 0 | 182 |
| GTT | 115 | GTT | 0 | GTT | 70 | 185 |
| TAA | 0 | TAA | 0 | TAA | 189 | 189 |
| TAC | 98 | TAC | 0 | TAC | 88 | 186 |
| TAG | 0 | TAG | 189 | TAG | 0 | 189 |

Table 2 (continued)

| $w^{\prime}$ in $\mathrm{q}=0$ | Number | $w^{\prime}$ in $\mathrm{q}=1$ | N umber | $w^{\prime}$ in $\mathrm{q}=2$ | N umber | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TAT | 1 | TAT | 0 | tat | 185 | 186 |
| TCA | 0 | TCA | 156 | TCA | 14 | 170 |
| TCC | 0 | TCC | 105 | TCC | 78 | 183 |
| TCG | 0 | TCG | 189 | TCG | 0 | 189 |
| TCT | 23 | TCT | 51 | TCT | 80 | 154 |
| TGA | 0 | TGA | 30 | TGA | 154 | 184 |
| TGC | 0 | TGC | 91 | TGC | 95 | 186 |
| TGG | 0 | TGG | 170 | TGG | 12 | 182 |
| TGT | 0 | TGT | 2 | TGT | 184 | 186 |
| TTA | 2 | TTA | 180 | TTA | 0 | 182 |
| TTC | 120 | TTC | 16 | TTC | 16 | 152 |
| TTG | 0 | TTG | 189 | TTG | 0 | 189 |
| TTT | 69 | TTT | 0 | TTT | 78 | 147 |
| (b) Eukaryotic protein coding genes |  |  |  |  |  |  |
| AAA | 187 | AAA | 0 | AAA | 0 | 187 |
| AAC | 187 | AAC | 2 | AAC | 0 | 189 |
| AAG | 183 | AAG | 6 | AAG | 0 | 189 |
| AAT | 189 | AAT | 0 | AAT | 0 | 189 |
| ACA | 0 | ACA | 189 | ACA | 0 | 189 |
| ACC | 108 | ACC | 60 | ACC | 3 | 171 |
| ACG | 0 | ACG | 189 | ACG | 0 | 189 |
| ACT | 68 | ACT | 118 | ACT | 0 | 186 |
| AGA | 0 | AGA | 109 | AGA | 77 | 186 |
| AGC | 0 | AGC | 166 | AGC | 19 | 185 |
| AGG | 0 | AGG | 189 | AGG | 0 | 189 |
| AGT | 2 | AGT | 105 | AGT | 74 | 181 |
| ATA | 0 | ATA | 189 | ATA | 0 | 189 |
| ATC | 189 | ATC | 0 | ATC | 0 | 189 |
| ATG | 0 | ATG | 189 | ATG | 0 | 189 |
| ATT | 189 | ATT | 0 | ATT | 0 | 189 |
| CAA | 1 | CAA | 0 | CAA | 188 | 189 |
| CAC | 0 | CAC | 2 | CAC | 187 | 189 |
| CAG | 124 | CAG | 61 | CAG | 1 | 186 |
| CAT | 0 | CAT | 0 | CAT | 189 | 189 |
| CCA | 3 | CCA | 186 | CCA | 0 | 189 |
| CCC | 15 | CCC | 75 | CCC | 55 | 145 |
| CCG | 0 | CCG | 189 | CCG | 0 | 189 |
| CCT | 27 | CCT | 0 | CCT | 155 | 182 |
| CGA | 0 | CGA | 0 | CGA | 189 | 189 |
| CGC | 7 | CGC | 0 | CGC | 182 | 189 |
| CGG | 0 | CGG | 1 | CGG | 188 | 189 |
| CGT | 0 | CGT | 0 | CGT | 189 | 189 |
| CTA | 0 | CTA | 97 | CTA | 92 | 189 |
| CTC | 61 | CTC | 60 | CTC | 39 | 160 |
| CTG | 95 | CTG | 94 | CTG | 0 | 189 |
| CTT | 0 | CTT | 0 | CTT | 189 | 189 |
| GAA | 96 | GAA | 0 | GAA | 93 | 189 |
| GAC | 189 | GAC | 0 | GAC | 0 | 189 |
| GAG | 189 | GAG | 0 | GAG | 0 | 189 |
| GAT | 188 | GAT | 0 | GAT | 1 | 189 |
| GCA | 48 | GCA | 121 | GCA | 8 | 177 |
| GCC | 189 | GCC | 0 | GCC | 0 | 189 |

Table 2 (continued)

| $\mathrm{w}^{\prime}$ in $\mathrm{q}=0$ | Number | $w^{\prime}$ in $\mathrm{q}=1$ | Number | $w^{\prime}$ in $q=2$ | N umber | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| GCG | 0 | GC G | 189 | GCG | 0 | 189 |
| GCT | 118 | GCT | 3 | GCT | 65 | 186 |
| GGA | 1 | GGA | 0 | GGA | 188 | 189 |
| GGC | 108 | GGC | 0 | GGC | 67 | 175 |
| GGG | 0 | G G G | 0 | GGG | 189 | 189 |
| GGT | 101 | G G T | 0 | GGT | 88 | 189 |
| GTA | 4 | GTA | 130 | GTA | 53 | 187 |
| GTC | 189 | GTC | 0 | GTC | 0 | 189 |
| GTG | 117 | GTG | 72 | GTG | 0 | 189 |
| GTT | 131 | GTT | 0 | GTT | 54 | 185 |
| TAA | 0 | TAA | 0 | TAA | 189 | 189 |
| TAC | 188 | TAC | 0 | TAC | 1 | 189 |
| TAG | 0 | TAG | 189 | TAG | 0 | 189 |
| TAT | 94 | TAT | 0 | TAT | 84 | 178 |
| TCA | 0 | TCA | 189 | TCA | 0 | 189 |
| TCC | 6 | TCC | 131 | TCC | 40 | 177 |
| TCG | 0 | TCG | 189 | TCG | 0 | 189 |
| TCT | 70 | TCT | 101 | TCT | 0 | 171 |
| TGA | 0 | TGA | 0 | TGA | 189 | 189 |
| TGC | 2 | TGC | 45 | TGC | 140 | 187 |
| TGG | 0 | TGG | 141 | TGG | 47 | 188 |
| TGT | 0 | TGT | 0 | TGT | 189 | 189 |
| TTA | 1 | TTA | 188 | TTA | 0 | 189 |
| TTC | 189 | TTC | 0 | TTC | 0 | 189 |
| TTG | 0 | TTG | 189 | TTG | 0 | 189 |
| TTT | 106 | TTT | 0 | TTT | 73 | 179 |

The last column gives the total number of classifiable autocorrelation functions. The trinucleotides in bold have a preferential occurrence frame.
(a) The trinucleotides in italics, classified into two frames (CTA, GTA, GTG, TCT, TGC, TTT) or misclassified ( $<50$ : GCA, TG G), have been assigned to the frame according to the properties identified with the other trinucleotides, both in prokaryotes and eukaryotes.
(b) The trinucleotides in italics, classified into two frames (AGA, AGT, CTA, GCT, GTG, TAT) or misclassified ( $<50$ : AAG, GCA, GTA, TGC, TGG), have been assigned to the frame according to the properties identified with the other trinucleotides, both in prokaryotes and eukaryotes.

### 3.3. Complementarity property

Recall of the DNA complementarity rule (Watson and Crick, 1953): (i) the DNA double helix consists of two nucleotide sequences $s_{1}$ and $s_{2}$ connected with the nucleotide pairing (hydrogen bonds) according to the complementarity rule $\mathscr{C}$ : the nucleotide A (resp. C, G, T) in $\mathrm{s}_{1}$ pairs with the complementary nucleotide $\mathscr{C}(\mathrm{A})=\mathrm{T} \quad($ resp. $\mathscr{C}(\mathrm{C})=\mathrm{G}, \mathscr{C}(\mathrm{G})=\mathrm{C}, \mathscr{C}(\mathrm{T})=\mathrm{A})$ in $\mathrm{s}_{2}$; (ii) the two nucleotide sequences $s_{1}$ and $s_{2}$ run in opposite directions (called antiparallel) in the DNA double helix: the trinucleotide $w=I_{1} I_{2} I_{3}, I_{1}, I_{2}, I_{3} \in\{A, C, G$,

T \}, in $\mathrm{s}_{1}$ pairs with the complementary trinucleotide $\mathscr{C}(w)=\mathscr{C}\left(I_{3}\right) \mathscr{C}\left(I_{2}\right) \mathscr{C}\left(I_{1}\right)$ in $s_{2}$.

Property 2: $\mathscr{C}\left(\mathrm{T}_{0}\right)=\mathrm{T}_{0}, \quad \mathscr{C}\left(\mathrm{~T}_{1}\right)=\mathrm{T}_{2} \quad$ and $\mathscr{C}\left(\mathrm{T}_{2}\right)=\mathrm{T}_{1}$ ( Table 3(b)). $\mathrm{T}_{0}$ is self-complementary (11 trinucleotides of $\mathrm{T}_{0}$ are complementary to 11 other trinucleotides of $T_{0}$ ) and, $T_{1}$ and $T_{2}$ are complementary to each other (the 21 trinucleotides of $T_{1}$ are complementary to the 21 trinucleotides of $\left.\mathrm{T}_{2}\right)$. Note also that $\mathscr{C}\left(\mathrm{X}_{0}\right)=\mathrm{X}_{0}$, $\mathscr{C}\left(\mathrm{X}_{1}\right)=\mathrm{X}_{2}$ and $\mathscr{C}\left(\mathrm{X}_{2}\right)=\mathrm{X}_{1}$.

Biological consequence (detailed in the discussion): the two paired reading frames may simultaneously code for amino acids.
Table 3

(a) List of the trinucleotides per frame in lexicographical order deduced from the Table $2(a-b)$. Three subsets of trinucleotides can be identified: $T_{0}=X_{0} \cup\{A A A$, TTT $\}$ in frame $0, T_{1}=X_{1} \cup\{C C C\}$ in frame 1 and $T_{2}=X_{2} \cup\{G G G\}$ in frame 2.
(b) Complementarity property with the three subsets $T_{0}, T_{1}$ and $T_{2}$ of trinucleotides identified in Table 3 (a).
(c) Circularity property with the three subsets $X_{0}, X_{1}$ and $X_{2}$ of trinucleotides identified in Table 3(a).
(d) Circular code statistics in the al phabets $\{\mathrm{R}, \mathrm{Y}\}$ and $\{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$.

### 3.4. Circularity property

Definition of the trinucleotide circular permutation: the circular permutation $\mathscr{P}$ of the trinucleotide $w=I_{1} I_{2} I_{3}, I_{1}, I_{2}, I_{3} \in\{A, C, G, T\}$, is the permutated trinucleotide $\mathscr{P}(\mathrm{w})=\mathrm{I}_{2} \mathrm{I}_{3} \mathrm{I}_{1}$.

Property 3: $\mathscr{P}\left(\mathrm{X}_{0}\right)=\mathrm{X}_{1}$ and $\mathscr{P}\left(\mathrm{X}_{1}\right)=\mathrm{X}_{2}$ (Table $3(c)$ ). $X_{0}$ generates $X_{1}$ by one circular permutation and $X_{2}$ by another circular permutation (one and two circular permutations with each trinucleotide of $X_{0}$ lead to the trinucleotides of $X_{1}$ and $X_{2}$, respectively).

Biological consequence (detailed in the discussion): the two subsets $X_{1}$ and $X_{2}$ can be deduced from $X_{0}$.

### 3.5. Circular code property

### 3.5.1. Definition of a circular code

Recall of a few notations: let $\mathscr{B}$ be a genetic alphabet, $\mathscr{B}_{2}=\{\mathrm{R}, \mathrm{Y}\}$ and $\mathscr{B}_{4}=\{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$. $\mathscr{B}^{*}$ denotes the words on $\mathscr{B}$ of finite length including the empty word of length $0 . \mathscr{B}^{+}$denotes the words on $\mathscr{B}$ of finite length $\geq 1$. Let $w_{1} w_{2}$ be the concatenation of the two words $w_{1}$ and $w_{2}$.
A subset $X$ of $\mathscr{B}^{+}$is a circular code if for all $n, m \geq 1$ and $x_{1}, x_{2}, \ldots, x_{n} \in X, y_{1}, y_{2}, \ldots, y_{m} \in X$ and $p \in \mathscr{B}^{*}, \quad s \in \mathscr{B}^{+}$, the equalities $s x_{2} x_{3} \ldots x_{n} p=$ $y_{1} y_{2} \ldots y_{m}$ and $x_{1}=p s$ imply $n=m, p=1$ and $x_{i}=$ $y_{i}, 1 \leq i \leq n$ (Béal, 1993; Berstel and Perrin, 1985) (Fig. 2(a)). In other terms, every word on $\mathscr{B}$ 'written on a circle' has at most one factorization (decomposition) over $X$.
Remark: In the following, $X$ will be a set of words of length three as a protein gene is a concatenation of trinucleotides.
3.5.2. Complete study of circular codes with trinucleotides on the alphabet $\mathscr{B}_{2}=\{R, Y\}$

Such a complete study has not been presented so far and allows to introduce simply the differents concepts and properties of circular codes which are complex in the alphabet $\mathscr{B}_{4}=\{\mathrm{A}, \mathrm{C}, \mathrm{G}$, T\} and cannot be analyzed by hand.

If b is the cardinal of the alphabet $\mathscr{B}$, then X contains at most $b^{3}$ trinucleotides (Table 3(d)). Therefore, on $\mathscr{B}_{2}, \mathrm{X}$ is a subset of $\{R R R, R R Y$, RYR, RYY, YRR, YRY, YYR, YYY\}. There

(b)


Fig. 2. (a) A representation of the definition of a circular code; (b) flower automaton $\mathscr{F}\left(\mathrm{X}_{0}\right)$ associated with the circular code $X_{0}$.
are two obvious constraints so that $X$ is a circular code:
(i) $X$ cannot have the trinucleotides $w=I I I$, $I \in \mathscr{B}$. F or example, if $X$ contains $R R R$ then the word ... R R R R R R ... has three factorizations over $X: \ldots R R R, R R R \ldots, \ldots R, R R R, R R \ldots$ and $\ldots R R$, $R R R, R \ldots$ Therefore, $X$ will be a subset of $\mathscr{B}_{2}^{\prime}=$ $\{R R Y, R Y R, R Y Y, Y R R, Y R Y, Y Y R\}$. The cardinal of $\mathscr{B}^{\prime}$ is $b^{3}-\mathrm{b}$, i.e. six for $\mathscr{B}_{2}^{\prime}$.
(ii) X cannot have at the same time two trinucleotides deduced from each other by circular permutation. F or example, if $X$ contains $R R Y$ and $R Y R$ ( $R Y R$ is one circular permutation of $R R Y$ ) then the word ... RRYRRYRRY... has two factorizations over $X$ : ...RRY, RRY, RRY... and ... R, R Y R, R Y R , R Y ... Therefore, by gathering the six trinucleotides of $\mathscr{B}_{2}^{\prime}$ in two classes of three codons so that the three codons are deduced from each other by circular permutations $\{R R Y, R Y R$, $Y R R\}$ and $\{R Y Y, Y Y R, Y R Y\}, X$ has at most one trinucleotide in each class. Therefore, $X$ contains at most two trinucleotides and $3^{2}=9$ sets $X$ are potential (maximal) circular codes. The number of classes invariant by circular permutation is $\mathrm{Card}\left(\mathscr{B}^{\prime}\right) / 3=\left(\mathrm{b}^{3}-\mathrm{b}\right) / 3$, i.e. two on $\mathscr{B}_{2}$ (Table 3(d)). The number of potential (maximal) circular codes is $3^{\left(b^{3}-b\right) / 3}$, i.e. $3^{2}=9$ on $\mathscr{B}_{2}$ (Table $3(d)$ ).
$X_{a}=\{R R Y, R Y Y\}=R N Y$ is a circular code (demonstrated in the introduction). $X_{a}$ is a maximal (two trinucleotides) circular code and corresponds to the RNY model (Eigen and Schuster, 1978). $X_{a}$ is self-complementary (complementary circular code), i.e. $\mathscr{C}\left(X_{a}\right)=X_{a}$, as RRY and RYY are complementary. A ny subset of $X_{a}$ is also a circular code but not maximal. F or example, the subset RRY is a non-maximal circular code and corresponds to the RRY model (Crick et al., 1976). The two subsets $X_{b}=\mathscr{P}\left(X_{a}\right)=\{R Y R$, Y Y R \} and $X_{c}=\mathscr{P}\left(X_{b}\right)=\{Y R R, Y R Y\}$ obtained by circular permutations of $X_{a}$ are also maximal circular codes (identical proof). $X_{b}$ and $X_{c}$ are complementary to each other, i.e. $\mathscr{C}\left(X_{b}\right)=X_{c}$ and $\mathscr{C}\left(X_{c}\right)=X_{b}$, as R Y R (resp. Y Y R ) and Y R Y (resp. $Y R R$ ) are complementary.

The previous results remain unchanged by substituting $R$ by $Y$ and reciprocally. Therefore, $X_{d}=$ $\{Y R R, \quad Y Y R\}$ is a maximal complementary circular code $\left(\mathscr{C}\left(X_{d}\right)=X_{d}\right)$ whose two subsets
$X_{e}=\mathscr{P}\left(X_{d}\right)=\{R R Y, \quad Y R Y\} \quad$ and $\quad X_{f}=\mathscr{P}\left(X_{e}\right)=$ \{RYR, R Y Y \} obtained by circular permutations of $X_{d}$ are also maximal circular codes and complementary to each other $\left(\mathscr{C}\left(X_{e}\right)=X_{f}\right.$ and $\mathscr{C}\left(X_{f}\right)=$ $X_{e}$ ).

The three remaining sets $X$ are $X_{g}=\{R Y Y$, $Y R R\}$ and $X_{h}=\mathscr{P}\left(X_{g}\right)=\mathscr{C}\left(X_{g}\right)=\{R R Y, Y Y R\}$ which are circular codes and $X_{i}=\mathscr{P}\left(X_{h}\right)=\{R Y R$, YRY \} which is not a circular code as the word ... RYRYRYRYR... has two factorizations over $X_{i}: \ldots R Y R, Y R Y, R Y R \ldots$, and $\ldots R, Y R Y, R Y R$, Y R ...

In summary, on $\mathscr{B}_{2}$, eight among nine sets $X$ are circular codes and two sets $X_{a}=\{R R Y, R Y Y\}=$ $R N Y$ and $X_{d}=\{Y R R, Y Y R\}=Y N R$ are complementary Circular codes with two permutated Circular codes (called $C^{3}$ codes) (Table 3(d)).
3.5.3. Identification of circular codes with trinucleotides on the alphabet $\mathscr{B}_{4}=\{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$

The study of circular codes on $\mathscr{B}_{4}$ is obviously more complex. For example, the search of a unique factorization needs the introduction of some classical definitions and results in coding theory, e.g. the flower automaton (Beal, 1993; Berstel and Perrin, 1985). The results obtained on $\{A, C, G, T\}$ are new. From a biological point of view, no circular code (code without commas) on $\{A, C, G, T\}$ has been identified with a theoretical, statistical or experimental approach. From a computational point of view, the classical circular codes determined by the methods of automatic construction of circular codes impose constraints on the choice of letters in the sites of the words, e.g. absence of a given letter in a given site of all words.

Property 4: the subset $X_{0}$ is a maximal (20 trinucleotides) circular code. The subsets $X_{1}$ and $X_{2}$ are also maximal circular codes. Remark: The property that $X_{0}$ is a circular code does not necessarily imply that $X_{1}$ and $X_{2}$ are also circular codes.

Proofs. (i) Proof of the maximum cardinal of a circular code. The 60 words of $\mathscr{B}_{4}^{\prime}=\{\mathrm{AAA}, \ldots$, TTT \}-\{AAA, CCC, GGG, TTT\} are gathered in $\mathrm{Card}\left(\mathscr{B}_{4}^{\prime}\right) / 3=20$ classes invariant by circular permutation (Table 3(d)). A circular code with words of length three on $\mathscr{B}_{4}$ has at most one word in each class and then contains at most 20 words.
(ii) Proof that $X_{0}$ is a circular code. As on $\mathscr{B}_{4}$ there are $3^{\left(b^{3}-b\right) / 3}=3^{20}=3486784401$ potential circular codes (Table 3(d)), the use of some theorems in coding theory are necessary for the development of algorithms for automatically determining the circular codes. These basic theorems are given, the algorithms written in Pascal will be described elsewhere.
Definition 1: a deterministic finite state automaton $\mathscr{A}$ is said to be local if an integer $n$ exists so that any two paths in $\mathscr{A}$ of the same length n and of the same associated word, have the same terminal state.
Lemma 1: If $\mathscr{A}$ is a strongly connected automaton, the two following properties are equivalent: $\mathscr{A}$ is local and $\mathscr{A}$ does not contain two cycles labelled with the same word (Béal, 1993).

Definition 2: the flower automaton $\mathscr{F}(\mathrm{X})$ associated with a subset $X$ of $\mathscr{B}^{+}$has a particular state (labelled 1 in Fig. 2(b)) and cycles issued from this state 1 and labelled by words of $X$.
Lemma 2: a finite subset $X$ of $\mathscr{B}^{+}$is a finite circular code if and only if the flower automaton $\mathscr{F}(\mathrm{X})$ is a local automaton (Béal, 1993).

Fig. 2(b) gives the flower automaton $\mathscr{F}\left(X_{0}\right)$ associated with $X_{0}$. To prove that ' $X_{0}$ is a circular code' is equivalent to prove that ' $\mathscr{F}\left(\mathrm{X}_{0}\right)$ is local', i.e. $\mathscr{F}\left(\mathrm{X}_{0}\right)$ does not contain two cycles labelled with the same word. This proof can be done by hand (rather tedious and not explained here) or by algorithm. The algorithm developed identifies automatically all possible subsets X of $\mathscr{B}^{+}$verifying the condition of circular code and allows statistics with circular codes (Section 3.7).
(iii) Proof that $X_{1}$ and $X_{2}$ are circular codes: similar to (ii) by constructing the flower automata $\mathscr{F}\left(\mathrm{X}_{1}\right)$ and $\mathscr{F}\left(\mathrm{X}_{2}\right)$ associated with $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$, respectively.

In summary, the properties 1,2 and 3 imply that $X_{0}$ is a complementary Circular code with two permutated Circular codes (called $\mathrm{C}^{3}$ code).

### 3.6. A utomatic frame determination property

Property 5: the length of the minimal window to automatically retrieve the frame with $X_{0}$ is equal to 13 nucleotides.

The problem to automatically determine the decomposition of a word into trinucleotides of $X_{0}$ is introduced with an example. Indeed, the unicity of such a decomposition is not obvious. For example, the word $w^{\prime}=A G G T A A T T A C C A ~ o f ~$ length 12 can be decomposed into trinucleotides of $X_{0}$ in two ways: AG, GTA, ATT, ACC, A (GTA, ATT, ACC $\in X_{0}$, Table 3(a)) or A, GGT, AAT, TAC, CA (GGT, AAT, TAC $\in \mathrm{X}_{0}$, Table 3(a)). In fact, the automatic frame determination property is a consequence of the circular code property. If a word is constructed by concatenating words of $X_{0}$ and if the frame of construction is lost, then the property of code assures that the frame can be retrieved according to a unique way. Such a decomposition is called in the following the reading frame of the word according to the code $X_{0}$.

The unicity of such a decomposition is proved by using the properties of the associated flower automaton $\mathscr{F}\left(\mathrm{X}_{0}\right)$. As all words of $\mathrm{X}_{0}$ have a length of three (Fig. 2(b)), the states of the automaton $\mathscr{F}\left(\mathrm{X}_{0}\right)$ can be associated with frames, state 1 with frame 0 , states 2-5 with frame 1 and states 6-16 with frame 2 . If any letter of a word obtained by a concatenation of trinucleotides of $X_{0}$, can be associated with a unique state of the automaton $\mathscr{F}\left(\mathrm{X}_{0}\right)$, then a frame can be deduced for this letter because the associated unique state of $\mathscr{F}\left(\mathrm{X}_{0}\right)$ is related to a given frame. As a consequence, the word can be decomposed into trinucleotides of $X_{0}$ : its reading frame according to $X_{0}$ is then retrieved. Then, the problem consists in identifying such a unique state for a letter of the word. Such a unicity is not obvious. In the previous example, the factor $w^{\prime}=$ AGGTAATTACCA of length 12 can be attributed to two reading frames according to $X_{0}$ : frame 1 (initial state 3 or 4 of $w^{\prime}$ in $\mathscr{F}\left(X_{0}\right)$ ) or frame 2 (initial state 11 or 14 of $w^{\prime}$ in $\left.\mathscr{F}\left(X_{0}\right)\right)$ (Fig. 3(a)).

In the case of a local automaton $\mathscr{A}$, definition 1 asserts that there exists $n$ so that for a word of length $\geq \mathrm{n}$, all paths associated with this word have the same terminal state and thus, the reading frame of the word according to $X_{0}$ can be determined. For $\mathscr{A}=\mathscr{F}\left(\mathrm{X}_{0}\right), \mathrm{n}=13$ and $\mathscr{F}\left(\mathrm{X}_{0}\right)$ is called a 13-local automaton. In other words, the length of the minimal window to retrieve always
(a)

(b)


Fig. 3. (a) A $n$ example of the automatic frame determination of a word of length $\geq 13$ with the flower automaton $\mathscr{F}\left(X_{0}\right)$ (Fig. 2(b)). The states 1 are associated with the position of the commas. The word $w^{\prime}$ can be decomposed into trinucleotides of $X_{0}$ in two ways. The word $w=w^{\prime} A$ has a unique decomposition into trinucleotides of $X_{0}$; (b) examples of three concatenations of two trinucleotides of the $C^{3}$ code $X_{0}$ generating expected and unexpected trinucleotides in the shifted frames 1 and 2.
the frame is 13 letters with $X_{0}$. In the previous example, the first letter of the word $w=w^{\prime} A$ of length 13 is attributed to the unique frame, the frame 1 , as there is no edge labelled A leaving the state 9 of $\mathscr{F}\left(\mathrm{X}_{0}\right)$ (Fig. 2(b) and Fig. 3(a)). Then, the unique decomposition of $w$ according to $X_{0}$ is AG, GTA, ATT, ACC, AA (Fig. 3(a)).
The length n of the minimal window to automatically retrieve the reading frame of a word according to $\mathrm{X}_{0}$ can be determined by hand (rather tedious) or by algorithm testing all possible paths in the automaton. This computational approach allows statistics with the different $\mathrm{C}^{3}$ codes (Section 3.8).
Remark: The lengths of the minimal windows to automatically retrieve the frames 1 and 2 with
$X_{1}$ and $X_{2}$, respectively, are equal to 13 nucleotides (similar proof).

Biological consequence (detailed in the discussion): the code $X_{0}$ can retrieve automatically the frame 0 in any region of a protein gene (formed by a series of trinucleotides of $X_{0}$ ) without a start codon.

### 3.7. Rarity property

Property 6: the occurrence probability of $X_{0}=$ $6.2 \times 10^{-8}$.

Statistics concerning circular codes with trinucleotides on the 4-letter alphabet $\{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$ allow to determine the occurrence probability of the code $X_{0}$. There are $3^{20}=3486784401$ poten-
tial circular codes (Section 3.5). The number of circular codes computed with an algorithm verifying the definition of circular code among the $3^{20}$ circular codes, is 12964440 . The computed number of circular codes having the complementarity property (complementary circular codes), is 528. The computed number of complementary Circular codes with two permutated Circular codes ( $\mathrm{C}^{3}$ codes), is 216 . Therefore, the probability to have a $C^{3}$ code, e.g. $X_{0}$, is $216 / 3^{20}=6.2 \times 10^{-8}$. This very low probability explains the difficulty in identifying such $\mathrm{C}^{3}$ codes by hand or by the classical methods of automatic construction of circular codes.
Table 3(d) summarizes the circular code statistics on the alphabets $\{\mathrm{R}, \mathrm{Y}\}$ and $\{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$.

Biological consequence: the probability to observe the code $X_{0}$ in protein genes is very low and non-random.

### 3.8. Concatenation properties

The $C^{3}$ code $X_{0}$ identified in protein genes has some concatenation properties (flexibility) compared to the other $\mathrm{C}^{3}$ codes:

Property 7: $X_{0}$ has the maximum length (13 nucleotides) of the minimal window among the $216 C^{3}$ codes.

The lengths of minimal windows to retrieve the frame in the $216 \mathrm{C}^{3}$ codes are 5, 7, 9 and 13 nucleotides (data not shown), and equal to 13 nucleotides for $X_{0}$ (Section 3.6).

Property 8: $X_{0}$ has a high frequency ( $27.5 \%$ ) of misplaced trinucleotides in the shifted frames among the $216 C^{3}$ codes.
The circular permutation property implies that the concatenation of two trinucleotides of $X_{0}$ generates with a high probability a trinucleotide of $X_{1}$ in frame 1 and a trinucleotide of $X_{2}$ in frame 2. For $X_{0}$, this property is verified at $72.5 \%$, i.e. there are $27.5 \%$ of misplaced trinucleotides in the shifted frames. The concatenation of two identical trinucleotides (process called duplication in biology) of $X_{0}$ leads obviously to the expected trinucleotides in the shifted frames (e.g. $\mathrm{CAG} \in \mathrm{X}_{0}$ and $C A G \in X_{0}$ generate $A G C \in X_{1}$ in frame 1 and $G C A \in X_{2}$ in frame 2) (first example in Fig. 3(b)). However, the probability of this type of concate-
nation is too low $(1 / 20)$ to explain the circularity property. The concatenation of two different trinucleotides of $X_{0}$ may lead to the expected trinucleotides in the shifted frames (e.g. $C A G \in X_{0}$ and $G A G \in X_{0}$ generate $A G G \in X_{1}$ in frame 1 and $G G A \in X_{2}$ in frame 2) or not ( $G G T \in X_{0}$ and $A C C \in X_{0}$ generate $G T A \notin X_{1}$ in frame 1 and TAC $\notin X_{2}$ in frame 2) (second and third examples in Fig. 3(b)). Due to the complementarity property, the frequency of trinucleotides of $X_{0}$ (resp. $X_{2}$ ) found in the shifted frame 1 is equal to the frequency of trinucleotides of $X_{0}$ (resp. $X_{1}$ ) found in the shifted frame 2. Therefore, the frequencies of misplaced trinucleotides ( $\mathrm{X}_{0}$ and $\mathrm{X}_{2}$ ) found in the shifted frame 1 are equal to the frequencies of misplaced trinucleotides ( $\mathrm{X}_{0}$ and $\mathrm{X}_{1}$ ) found in the shifted frame 2.
Fig. 4 shows the repartition function of the 216 $\mathrm{C}^{3}$ codes according to the frequency of misplaced trinucleotides in frame 1 (or 2) generated by the concatenation of trinucleotides of a given $\mathrm{C}^{3}$ code. This frequency is between $6.5 \%$ and $31 \%$, and equal to $27.5 \%$ for $\mathrm{X}_{0}$.

Property 9: $X_{0}$ has an occurrence of the four types of nucleotides in the three trinucleotide sites. This structure explains that the code $X_{0}$ cannot be generated by the classical methods of automatic construction of circular codes.

Biological consequence (detailed in the discussion): the code $X_{0}$ has evolutionary properties.

## 4. Discussion

### 4.1. Simulation of the protein coding genes

As the code $X_{0}$ leads to $72.5 \%$ well-placed trinucleotides in the shifted frames (Fig. 4), the protein genes could be simulated uniquely with the trinucleotides in frame 0 (codons). In order to verify this hypothesis, a population S of 200 simulated genes of 1000 base length is generated by an independent mixing of the 22 codons of $T_{0}$ with equiprobability. A sample of 200000 bases allows precise and stable computations (i.e. there is no random fluctuations in the probability calculus of i-motifs: a sample having, for example, 50 simulated genes of 1000 base length leads to similar


Frequency (\%) of misplaced trinucleotides in frame 1 or 2
Fig. 4. Repartition function of the $216 C^{3}$ codes according to the frequency of misplaced trinucleotides in frame 1 or 2 generated by the concatenation of trinucleotides of a given $C^{3}$ code. The frequency of misplaced trinucleotides for the $C^{3}$ code $X_{0}=27.5 \%$.
results). The 12288 autocorrelation functions are computed in this simulated population S as defined in Section 2.1 and classified in the three types of periodicities according to the method given in Section 3.1.

For each trinucleotide $w^{\prime}$ and for each frame $q$ in the simulated population S, Table 4, similarly to the Table 2(a),(b), gives the number of classifiable autocorrelation functions $w^{p}(N)_{i} w^{\prime}$ implying the frame $q$ for $w^{\prime}$. This simple simulation retrieves the three types of periodicities correctly associated with the 12288 i-motifs and thus, the two other subsets $T_{1}$ and $T_{2}$ of trinucleotides in the frames 1 and 2 , respectively (except for three trinucleotides). $N$ ote: as this simulation uses only 22 trinucleotides, the maximal number of classifiable autocorrelation functions in the simulated genes (110) is obviously less than the one in the real protein genes (189).
4.2. Consequences on the purine/pyrimidine alphabet

The three subsets $T_{0}, T_{1}$ and $T_{2}$ classify the 64 $\mathrm{A} / \mathrm{C} / \mathrm{G} / \mathrm{T}$ trinucleotides according to their prefer-
ential occurrence frame. Therefore, a preferential occurrence frame for the eight R/Y trinucleotides can be deduced from the frames of the $64 \mathrm{~A} / \mathrm{C} / \mathrm{G} /$ T trinucleotides by considering for each $R / Y$ trinucleotide, the average frame of the eight frames associated with the eight $\mathrm{A} / \mathrm{C} / \mathrm{G} / \mathrm{T}$ specified trinucleotides. Table 5 shows that the subset $\mathscr{Y}_{0}=$ $\{R R Y, R Y Y\}=R N Y$ occurs preferentially in frame 0 , the subset $\mathscr{\mathscr { V }}_{1}=\{R Y R, Y Y R\}$, in frame 1 , and the subset $\mathscr{Y}_{2}=\{Y R R, Y R Y\}$, in frame 2. RRY and RYY have the same number (six) of A/C/G/T trinucleotides in frame 0 . The subset $\mathscr{Y}_{0}$ contains a few $\mathrm{A} / \mathrm{C} / \mathrm{G} / \mathrm{T}$ trinucleotides in frames 1 $\left(A C T^{1}, A G C^{1}\right)$ and $2\left(\mathrm{AGT}^{2}, \mathrm{GCT}^{2}\right)$ and $\mathscr{Y}_{1}$ and $\mathscr{Y}_{2}$, a few $A / C / G / T$ trinucleotides in frame 0 $\left(\mathrm{CAG}^{0}, \mathrm{CTG}^{0}, \mathrm{GTA}^{0}, \mathrm{TAC}^{0}\right)$. The subset $\mathscr{Y}_{0}$ is a $C^{3}$ code on the alphabet $\{R, Y\}$ (Section 3.5.2) and corresponds to the R $N$ Y codon model (Eigen and Schuster, 1978). There is no preferential frame for RRR and YYY.

These results also explain different previous works analyzing simultaneously the three frames (average frame) in protein genes with autocorrelation functions on the alphabet $\{R, Y\}$, in particular: (i) the periodicity 0 modulo 3 with the

Table 4
For each trinucleotide $w^{\prime}$ and for each frame $q$ in the simulated genes by an independent mixing of the 22 trinucleotides of $T_{0}$, the number of classifiable autocorrelation functions $\mathrm{w}^{\mathrm{p}}(\mathrm{N})_{\mathrm{i}} \mathrm{w}^{\prime}$ implying the frame q for $\mathrm{w}^{\prime}$ is given

| $\begin{aligned} & w^{\prime} \text { in } \\ & q=0 \end{aligned}$ | N umber | $\begin{aligned} & \mathrm{w}^{\prime} \text { in } \\ & \mathrm{q}=1 \end{aligned}$ | N umber | $\begin{aligned} & w^{\prime} \text { in } \\ & q=2 \end{aligned}$ | N umber |
| :---: | :---: | :---: | :---: | :---: | :---: |
| AAA | 110 | AAA | 0 | AAA | 0 |
| AAC | 110 | AAC | 0 | AAC | 0 |
| AAG | 0 | AAG | 110 | AAG | 0 |
| AAT | 110 | AAT | 0 | AAT | 0 |
| ACA | 0 | ACA | 110 | ACA | 0 |
| ACC | 110 | ACC | 0 | ACC | 0 |
| ACG | 0 | ACG | 110 | ACG | 0 |
| ACT | 0 | ACT | 92 | ACT | 0 |
| AGA | 0 | AGA | 2 | AGA | 0 |
| AGC | 0 | AGC | 97 | AGC | 0 |
| AGG | 0 | AGG | 110 | AGG | 0 |
| AGT | 0 | AGT | 0 | AGT | 82 |
| ATA | 0 | ATA | 110 | ATA | 0 |
| ATC | 110 | ATC | 0 | ATC | 0 |
| ATG | 0 | ATG | 110 | ATG | 0 |
| ATT | 110 | ATT | 0 | ATT | 0 |
| CAA | 0 | CAA | 0 | CAA | 110 |
| CAC | 0 | CAC | 0 | CAC | 110 |
| CAG | 110 | CAG | 0 | CAG | 0 |
| CAT | 0 | CAT | 0 | CAT | 110 |
| CCA | 0 | CCA | 25 | CCA | 0 |
| CCC | 0 | CCC | 110 | CCC | 0 |
| CCG | 0 | CCG | 110 | CCG | 0 |
| CCT | 0 | CCT | 0 | CCT | 110 |
| CGA | 0 | CGA | 0 | CGA | 110 |
| CGC | 0 | CGC | 0 | CGC | 110 |
| CGG | 0 | CGG | 0 | CGG | 110 |
| CGT | 0 | CGT | 0 | CGT | 110 |
| CTA | 0 | CTA | 0 | CTA | 110 |
| CTC | 110 | CTC | 0 | CTC | 0 |
| CTG | 110 | CTG | 0 | CTG | 0 |
| CTT | 0 | CTT | 0 | CTT | 110 |
| GAA | 110 | GAA | 0 | GAA | 0 |
| GAC | 110 | GAC | 0 | GAC | 0 |
| GAG | 110 | GAG | 0 | GAG | 0 |
| GAT | 110 | GAT | 0 | GAT | 0 |
| GCA | 0 | GCA | 86 | GCA | 0 |
| GCC | 110 | GCC | 0 | GCC | 0 |
| GCG | 0 | GCG | 110 | GCG | 0 |
| GCT | 0 | GCT | 0 | GCT | 90 |
| GGA | 0 | GGA | 0 | GGA | 110 |
| GGC | 110 | GGC | 0 | GGC | 0 |
| GGG | 0 | G G G | 0 | GGG | 110 |
| G G T | 110 | G GT | 0 | GGT | 0 |
| GTA | 110 | GTA | 0 | GTA | 0 |
| GTC | 110 | GTC | 0 | GTC | 0 |
| GTG | 0 | GTG | 110 | GTG | 0 |
| GTT | 110 | GTT | 0 | GTT | 0 |
| TAA | 0 | TAA | 0 | TAA | 110 |

Table 4 (continued)

| $\begin{aligned} & w^{\prime} \text { in } \\ & q=0 \end{aligned}$ | N umber | $\begin{aligned} & w^{\prime} \text { in } \\ & q=1 \end{aligned}$ | N umber | $\begin{aligned} & w^{\prime} \text { in } \\ & q=2 \end{aligned}$ | Number |
| :---: | :---: | :---: | :---: | :---: | :---: |
| TAC | 110 | TAC | 0 | TAC | 0 |
| TAG | 0 | TAG | 110 | TAG | 0 |
| TAT | 0 | TAT | 0 | TAT | 110 |
| TCA | 0 | TCA | 110 | TCA | 0 |
| TCC | 0 | TCC | 110 | TCC | 0 |
| TCG | 0 | TCG | 110 | TCG | 0 |
| TCT | 0 | TCT | 18 | TCT | 0 |
| TGA | 0 | TGA | 0 | TGA | 110 |
| TGC | 0 | TGC | 0 | TGC | 86 |
| TGG | 0 | TGG | 0 | TGG | 43 |
| TGT | 0 | TGT | 0 | TGT | 110 |
| TTA | 0 | TTA | 110 | TTA | 0 |
| TTC | 110 | TTC | 0 | TTC | 0 |
| TTG | 0 | TTG | 110 | TTG | 0 |
| TTT | 110 | TTT | 0 | TTT | 0 |

The trinucleotides in bold have a preferential occurrence frame. The three trinucleotides in italics (A GA, GCA, TGC) are reclassified according to the properties of the $C^{3}$ code $X_{0}$.
autocorrelation function $Y R Y(N)_{i} Y R Y$ (Fig. 5(a), A rquès and Michel, 1987, 1994) as the trinucleotides YRY occurring preferentially in frame 2 generate a number multiple of three of bases between them; and (ii) the absence of the periodicity 0 modulo 3 with the autocorrelation function RRR(N) iRRR (Fig. 5(b), A rquès and Michel, 1993) as RRR does not occur in a preferential frame.

### 4.3. C onsequences on the genetic code

The codon subset $T_{0}$ codes for 13 amino acids (AA ): A la, A sn, A sp, G In, G lu, Gly, Ile, Leu, Lys, Phe, Thr, Tyr and Val (Table 6). As the subset $\mathrm{T}_{0}$ has 22 codons, several codons of $T_{0}$ code for the same AA. Almost all classes of AA are coded by $\mathrm{T}_{0}$ : the simplest ( G ly), aliphatic ( $\mathrm{Ala}, \mathrm{Ile}, \mathrm{Leu}, \mathrm{V}$ al), hydroxyl (Thr), acidic (A sp, Glu), amide (Asn, GIn), basic (Lys) and aromatic (Phe, Tyr) (two classes are not coded by $\mathrm{T}_{0}$ : the sulfur-containing A A: Cys and M et, and the cyclic AA: Pro). The 12 A A coded by $X_{0}$ are equally represented in the two classes of aminoacyl-tR N A synthetases with a class 1 associated with GIn, Glu, Ile, Leu, Tyr and Val, and a class 2 , with A la, A sn, A sp, G ly, Pheand Thr (rewieved in Schimmel et al., 1993; H artman, 1995; Saks and Sampson, 1995).
4.4. Consequences on the amino acid frequencies in proteins

There are seven amino acids (AA) which are not coded by the codon subset $\mathrm{T}_{0}$ : A rg, Cys, H is, M et, Pro, Ser and Trp (Table 6). Therefore, these seven AA should have the lowest frequencies in proteins. In order to verify this consequence of a code in protein genes, the frequencies of the 20 AA are computed in 9510 prokaryotic proteins ( 3044028 AA) and 20673 eukaryotic proteins ( 7521044 AA). They are obtained from the protein data base SWISS-PROT (released 29, June 1994). Then, these observed A A frequencies are compared with their expected AA frequencies (number of codons coding an AA divided by the total number of non-stop codons, i.e. 61). Table 7 shows that except for $M$ et, the six other $A A$ not coded by $T_{0}$, Arg, Cys, His, Pro, Ser and Trp, have the lowest observed/expected frequency ratios $(<0.8)$ in the proteins of both prokaryotes and eukaryotes. For Arg, a difference between the observed frequency and the frequency expected from the universal genetic code has already been mentioned (Jukes et al., 1975). M et is a particular case as the codon coding for $M$ et is also the start codon for establishing the frame 0 (reading frame) in actual genes.
In summary, both in prokaryotes and eukaryotes, there is a strong correlation between the usage of the codons of $T_{0}$ in protein genes and the amino acid frequencies in proteins.
4.5. Biological consequences which could be associated with the properties of the circular code

The identification of the same subset $X_{0}$ of 20 trinucleotides occurring preferentially (i.e. in comparison with $X_{1}$ and $X_{2}$ ) in frame 0 (reading frame) of two different taxonomic protein gene populations (prokaryotes and eukaryotes), suggests an evolution of protein genes according to two processes: a construction process of primitive protein genes followed by an evolutionnary process transforming the primitive protein genes into actual protein genes. In the model currently under investigation, the primitive protein genes are constructed of trinucleotides of $X_{0}$ (independent mix-
ing with equiprobability $1 / 20$ ), i.e. $X_{1}$ and $X_{2}$ do not occur in frame 0 . Two simple types of evolutionnary processes may generate $X_{1}$ and $X_{2}$ in frame 0 : (i) substitutions in the three sites of trinucleotides of $X_{0}$ or (ii) insertions and deletions of nucleotides (process named RNA editing) in the trinucleotides of $X_{0}\left(X_{1}\right.$ and $X_{2}$ are obtained in this case by circular permutation). There are several constraints associated with these random evolutionary processes, in particular the frequencies of $X_{1}$ and $X_{2}$ in frame 0 must remain less than the frequency of $X_{0}$ in frame 0 . Such a computational model based on an evolution of the number of trinucleotides (from 20 to 64), i.e. on an evolution of the number of amino acids coded by these trinucleotides (from 12 to 20 by using the hypothesis of the actual genetic code), may be related to several evolutionary biological models, in particular to the co-evolution of the aminoacyl-tR NA synthetases and the genetic code (W etzel, 1995).
Almost all actual protein genes (with $\mathrm{X}_{0}, \mathrm{X}_{1}$ and $X_{2}$ in frame 0 ) begin with the ATG start codon which does not belong to $X_{0}\left(A T G \in X_{1}\right)$. H owever, the scanning mechanism (K ozak, 1978) for initiation of translation in eukaryotes (40S ribosomal subunit carrying $M$ et-tRNAi met and various initiation factors) is based on the consensus sequence GCCGCCRCCATG ( $\mathrm{R}=\mathrm{A}$ or G ; K ozak, 1989). Surprisingly, the three trinucleotides preceding the ATG start codon belong to $X_{0}\left(A C C, G C C \in X_{0}\right)$. The occurrence probability of such a series is equal to $1 /(20)^{3} \approx 10^{-4}$. Therefore, this motif of nine base length could have been the translation initiation signal in primitive protein genes (with only $X_{0}$ in frame 0). The importance of this motif has been demonstrated by site-directed mutagenesis experiments (e.g. K ozak, 1986a) and confirmed by the discovery of a type of thalassemia in which a mutation in the sequence (the purine base R three bases before ATG changed in C) drastically impairs translation initiation of $\alpha$-globin (M orlé et al., 1985). NonATG start codons exist in actual protein genes but they are weakly recognized like the ATG start codons in a mutated consensus sequence. Surprisingly, some non-ATG start codons identified, belong to $X_{0}$, e.g. CTG in the first exon of c-myc

Table 5
The eight $R / Y$ trinucleotides $(R=$ purine $=A$ or $G, Y=$ pyrimidine $=C$ or $T)$ are associated with the $64 A / C / G / T$ trinucleotides by considering their frame $\left(T_{0}, T_{1}, T_{2}\right)$

| RRR | RRY | RYR | RYY | YRR | YRY | Y YR | YYY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A A A^{0}$ | $\mathrm{AAC}^{0}$ | $A^{\prime} A^{1}$ | $\mathrm{ACC}^{0}$ | $C A A^{2}$ | $\mathrm{CAC}^{2}$ | CCA ${ }^{1}$ | $\mathrm{CCC}^{1}$ |
| $A A G^{1}$ | $A A T^{0}$ | ACG ${ }^{1}$ | $\mathrm{ACT}^{1}$ | $C A G{ }^{0}$ | CAT ${ }^{2}$ | CCG ${ }^{1}$ | $\mathrm{CCT}^{2}$ |
| AGA ${ }^{2}$ | AGC ${ }^{1}$ | ATA ${ }^{1}$ | ATC ${ }^{0}$ | $C G A{ }^{2}$ | CGC ${ }^{2}$ | CTA ${ }^{2}$ | CTC ${ }^{0}$ |
| AGG ${ }^{1}$ | AGT ${ }^{2}$ | ATG ${ }^{1}$ | ATT ${ }^{0}$ | $C G G^{2}$ | CGT ${ }^{2}$ | CTG ${ }^{0}$ | $\mathrm{CTT}^{2}$ |
| GAA ${ }^{0}$ | GAC ${ }^{0}$ | GCA ${ }^{2}$ | GCC ${ }^{0}$ | TAA ${ }^{2}$ | TAC ${ }^{0}$ | TCA ${ }^{1}$ | TCC ${ }^{1}$ |
| GAG ${ }^{0}$ | GAT ${ }^{0}$ | GCG ${ }^{1}$ | $\mathrm{GCT}^{2}$ | TAG ${ }^{1}$ | TAT ${ }^{2}$ | TCG ${ }^{1}$ | TCT ${ }^{1}$ |
| GGA ${ }^{2}$ | GGC ${ }^{0}$ | GTA ${ }^{0}$ | GTC ${ }^{0}$ | TGA ${ }^{2}$ | TGC ${ }^{1}$ | TTA ${ }^{1}$ | TTC ${ }^{0}$ |
| G G G ${ }^{2}$ | $\mathrm{GGT}^{0}$ | GTG ${ }^{1}$ | GTT ${ }^{0}$ | TGG ${ }^{2}$ | TGT ${ }^{2}$ | TTG ${ }^{1}$ | TTT ${ }^{0}$ |
| 0, 1, 2 | 0 | 1 | 0 | 2 | 2 | 1 | 0, 1, 2 |

The last row gives the preferential occurrence frame for the $R / Y$ trinucleotides, e.g. RRR is in the three frames (three A/C/G/T trinucleotides in frame 0, two in frame 1, three in frame 2), Y RY is in frame 2 (one A/C/G/T trinucleotide in frame 0 , one in frame 1 , six in frame 2).
(G eballe and M orris, 1994), A TT in several mitochondrial protein genes of the gastropod mollusc Cepea nemoralis (Terrett et al., 1996) and in the mitochondrial protein genes NADH2 of Homo and chimpanzees (A rnason et al., 1996). F urthermore, the first ATG codon is not always used for translation initiation which can begin at downstream ATG codons in the following cases: (i) the length between the cap and the first ATG codon is less than nine bases (e.g. Strubin et al., 1986); (ii) the first ATG codon occurs in a mutated consensus sequence (K ozak, 1986b); and (iii) the first ATG codon is associated with alternative promoters and/or splice sites for regulating downstream ATG codons, e.g. in protooncogenes, growth factor genes and homeobox genes (K ozak, 1989; Geballe and M orris, 1994). Finally, the lengths of the consensus motif GCCGCCRCC and of the motif between the cap and the first ATG codon could be considered according to the window of nucleotides automatically retrieving the frame 0 with the identified $C^{3}$ code $X_{0}$ (see also below).

A protein gene formed by trinucleotides of $X_{0}$ has the property to automatically retrieve the frame 0 in any region of the gene without a start codon ( $\mathrm{X}_{0}$ is a circular code by definition). It would be interesting to know if a trace of this property may exist in the actual protein genes formed by a preferential occurrence of trinucle-
otides of $X_{0}$ (i.e. a protein gene formed by trinucleotides of $X_{0}, X_{1}$ and $X_{2}$ with lower frequencies for $X_{1}$ and $X_{2}$ ). Surprisingly, such a property is observed with a biological process, called translational frameshifting, which is involved in: (i) producing a translational fusion for the morphogenesis of the viral particle, e.g. the retroviral gag and pol genes (F arabaugh et al., 1993); e.g. the retrotransposon TYA and TYB genes (Belcourt and F arabaugh, 1990); (ii) regulating gene expression, e.g. the Escherichia coli prfB gene (e.g. Craigen and Caskey, 1986) and the rat ODCase antizyme gene (e.g. M atsufuji et al., 1995); and (iii) coding two proteins with a common N -terminal region and a different C-terminal region, e.g. the Escherichia coli dnaX gene (e.g. Tsuchihashi and Brown, 1992), and the prokaryotic insertion sequences, e.g. IS1 (M achida et al., 1984), IS150: (Vögele et al., 1991). This frameshifting process allows translation to continue through a stop codon in frame 0 , to change the frame and to use two frames. It is found in genes associated with different functions and from a variety of organisms: bacteria, lower eukaryotes (yeasts, plants), higher eukaryotes (animal) and viruses (e.g. retroviruses, bacteriophages, plant viruses, etc.). There are mainly two translational frameshifting processes: hopping and slipping.

Hopping can be defined as a translational shift $\geq 2$ bases in the $5^{\prime}-3^{\prime}$ direction (downstream di-
rection) or in the $3^{\prime}-5^{\prime}$ direction (upstream direction). It requires a take-off trinucleotide and a landing trinucleotide. O'Connor et al. (1989) reported two examples, both decoded as V al, where the take-off trinucleotide (underlined once) belongs to $X_{1}$ and the landing trinucleotide (underlined twice), to $X_{0}$ : GTGTA ( $G T G \in X_{1}, T G T \in X_{2}$, GTA $\in X_{0}$ ) and $\underline{G T G T A} A G T$ ( $G T G \in X_{1}$, $T G T \in X_{2}, \quad G T A \in X_{0}, \quad T A A \in X_{2}, \quad A A G \in X_{1}$, $A G T \in X_{2}, G T T \in X_{0}$ ). In the first example, the take-off trinucleotide GTG and TGT are not translated and belong to $X_{1}$ and $X_{2}$, respectively. The translated trinucleotide GTA belongs to $X_{0}$. The second example appears to be an extension of the rule applied previously. If the first landing trinucleotide belonging to $X_{0}$ (GTA) is not chosen, then the second landing trinucleotide considered is



Fig. 5. (a) periodicity 0 modulo 3 in the eukaryotic protein coding genes with the autocorrelation function YRY(N) $Y$ Y $Y$ without considering the frame. The horizontal axis represents the number $\mathrm{i}, \mathrm{i} \in[0,50]$, of any bases N between $Y R Y$ and Y R Y . The vertical axis represents the autocorrelation function YRY(N) YRY; (b) absence of the periodicity 0 modulo 3 in the eukaryotic protein coding genes with the autocorrelation function $R R R(N)_{\mathrm{i}} R R R$ without considering the frame. The horizontal axis represents the number $\mathrm{i}, \mathrm{i} \in[0,50]$, of any bases $N$ between RRR and RRR. The vertical axis represents the autocorrelation function $R R R(N)_{i} R R R$.
the next downstream trinucleotide belonging to $X_{0}$ (GTT).
Slipping can be defined as a translational shift of one base either in the $3^{\prime}-5^{\prime}$ direction ( -1 frameshift) or in the $5^{\prime}-3^{\prime}$ direction ( +1 frameshift). Several trinucleotides which may direct -1 frameshift, have been reported, e.g. A A G (E. coli dnaX gene: Tsuchihashi and Brown, 1992; Lindsley and Gallant, 1993; the prokaryotic insertion sequence IS150: Vögele et al., 1991), A GC (Bruce et al., 1986), CCG (D ayhuff et al., 1986) and TTA (J acks et al., 1988; ten D am et al., 1990). As these trinucleotides belong to $X_{1}$ and as $X_{1}$ is deduced by one circular permutation of $X_{0}$, the theoretical frame 0 is retrieved with -1 frameshift, as expected with the examples mentioned. In contrast, other trinucleotides, e.g. A G T ( $F$ arabaugh et al., 1993), CTT (W eiss et al., 1987) and TGA (Craigen and Caskey, 1986; Curran, 1993), may provoke +1 frameshift. As these trinucleotides belong to $X_{2}$ and as $X_{2}$ is deduced by two circular permutations of $X_{0}$, the theoretical frame 0 is retrieved with +1 frameshift, as expected with the examples given. The trinucleotides AAA (Weiss et al., 1990), GGG (Weiss et al., 1990) and TTT (F ox and W eiss-Brummer, 1980) may induce both -1 and +1 frameshifts. Therefore, the trinucleotides belonging to $T_{0}, T_{1}$ and $T_{2}$ exclusively ( $A A A, T T T \in T_{0}, G G G \in T_{2}$ ), may lead to a shift of one base in both directions.

The rules of the take-off and landing trinucleotides in the hopping process and of the frameshift trinucleotides in the slipping process could be analyzed according to the subsets $X_{0}, X_{1}$ and $X_{2}$ of trinucleotides in order to identify the frameshift rules encoded in the D N A sequence. As $X_{0}, X_{1}$ and $X_{2}$ are associated with the frame 0,1 and 2 , respectively, the type of translational frameshifting at the frameshift site could easily be determined for example, a gene in a shifted frame 1 or 2 retrieving the frame 0 , a gene in frame 0 keeping the frame 0 and a gene in frame 0 using a shifted frame 1 or 2 . Indeed, a feature classically used for determining the frame 0 after a frameshift site, is the frame among the three possible ones having the highest number of non-stop codons before a stop codon. A set of 20 trinucleotides would be more significant than a stop codon for discriminating the different frames.

Table 6
The subset $T_{0}$ (bold) codes for 13 amino acids in the universal genetic code: Ala, A sn, A sp, Gln, Glu, Gly, Ile, Leu, Lys, Phe, Thr, Tyr and Val

| Codon | A mino acid | Codon | A mino acid | Codon | A mino acid | Codon | A mino acid |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| tTt | Phe, F | TCT | Ser, S | $\underline{\underline{\text { TAT }}}$ | Tyr, Y | $\underline{\underline{\text { TGT }}}$ | Cys, C |
|  | Phenylalanine |  | Serine |  | Tyrosine |  | Cysteine |
| TTC | Phe, F | TCC | Ser, S | TAC | Tyr, Y | TGC | Cys, C |
|  | Phenylalanine |  | Serine |  | Tyrosine |  | Cysteine |
| TTA | Leu, L | TCA | Ser, S | $\underline{\underline{\text { TAA }}}$ | Stop codon | $\underline{\underline{\text { TGA }}}$ | Stop codon |
|  | Leucine |  | Serine |  | ochre |  | opal |
| TTG | Leu, L | TCG | Ser, S | TAG | Stop codon | $\underline{\underline{\text { TGG }}}$ | Trp, W |
|  | Leucine |  | Serine |  | amber |  | Tryptophan |
| $\underline{\underline{C T T}}$ | Leu, L | $\underline{\underline{C C T}}$ | Pro, P | CAT | His, H | $\underline{\underline{C G T}}$ | Arg, R |
|  | Leucine |  | Proline |  | Histidine |  | Arginine |
| CTC | Leu, L | CCC | Pro, P | $\underline{\underline{C A C}}$ | His, H | $\underline{\underline{C G C}}$ | Arg, R |
|  | Leucine |  | Proline |  | Histidine |  | Arginine |
| $\underline{\underline{\text { CTA }}}$ | Leu, L | CCA | Pro, P | $\underline{\underline{C A A}}$ | GIn, Q | $\underline{\underline{C G A}}$ | Arg, R |
|  | Leucine |  | Proline |  | Glutamine |  | Arginine |
| CTG | Leu, L | CCG | Pro, P | CAG | GIn, Q | $\underline{\underline{\text { CGG }}}$ | Arg, R |
|  | Leucine |  | Proline |  | Glutamine |  | Arginine |
| ATT | Ile, I | ACT | Thr, T | AAT | A sn, N | $\underline{\text { AGT }}$ | Ser, S |
|  | Isoleucine |  | Threonine |  | A sparagine |  | Serine |
| ATC | Ile, I | ACC | Thr, T | AAC | A sn, N | AGC | Ser, S |
|  | Isoleucine |  | Threonine |  | A sparagine |  | Serine |
| ATA | Ile, I | ACA | Thr, T | AAA | Lys, K | $\underline{\text { AGA }}$ | Arg, R |
|  | Isoleucine |  | Threonine |  | Lysine |  | A rginine |
| ATG | M et, M | ACG | Thr, T | AAG | Lys, K | A G G | Arg, R |
|  | $M$ ethionine |  | Threonine |  | L ysine |  | Arginine |
| GTT | Val, V | $\underline{\underline{\text { GCT }}}$ | Ala, A | GAT | Asp, D | GGT | Gly, G |
|  | Valine |  | Alanine |  | A spartic acid |  | Glycine |
| GTC | Val, V | GCC | Ala, A | GAC | A sp, D | GGC | Gly, G |
|  | Valine |  | Alanine |  | A spartic acid |  | Glycine |
| GTA | Val, V | GCA | Ala, A | GAA | Glu, E | $\underline{\underline{G G A}}$ | Gly, G |
|  | Valine |  | Alanine |  | Glutamic acid |  | Glycine |
| $\underline{\text { GTG }}$ | Val, V | GCG | Ala, A | GAG | Glu, E | $\underline{\underline{\text { GGG }}}$ | Gly, G |
|  | V aline |  | Alanine |  | Glutamic acid |  | Glycine |

Seven amino acids are not coded by $\mathrm{T}_{0}$ : Arg, Cys, His, M et, Pro, Ser and Trp. The subset $\mathrm{T}_{1}$ (underlined once) is associated with 11 amino acids: Ala, Arg, Cys, Ile, Leu, Lys, M et, Pro, Ser, Thr and Val. The subset $T_{2}$ (underlined twice) is associated with 11 amino acids: Ala, Arg, Cys, Gln, Gly, His, Leu, Pro, Ser, Trp and Tyr.

The translational frameshifting in the actual protein genes has been considered in the previous section according to the words of the circular code $X_{0}$. In this section, this frameshifting process is analyzed with respect to the window of 13 nucleotides retrieving automatically the frame 0 of the circular code $X_{0}$.
In the hopping process, the window of $X_{0}$ could be related to the sequence between the take-off and landing trinucleotides.

In the -1 slipping process of eukaryotes and viruses, a secondary structure, usually a pseudo-
knot, is often associated with frameshifting (ten Dam et al., 1990). Pseudoknots occur on average six nucleotides downstream $(+6)$ of the frameshift trinucleotides (ten Dam et al., 1990). This distance is critical as the insertion or deletion of two nucleotides between frameshift trinucleotides and pseudoknots eliminate frameshifting (Brierley et al., 1992). Furthermore, the four nucleotides upstream $(-4)$ of the frameshift trinucleotides can modify the rate of frameshifting (Brierley et al., 1992). Therefore, these biological observations identify a sequence of $13(4+3+6)$
Table 7
Arg, Cys, His, Pro, Ser and Trp have the lowest observed/expected frequency ratios ( $<0.8$ ) in the proteins of both prokaryotes ( 9510 sequences, 3044028 amino acids) and eukaryotes ( 20673 sequences, 7521044 amino acids) as expected with the usage of the codons of $\mathrm{T}_{\mathrm{o}}$ in protein genes

| Amino acid (number of codons) | Expected frequency (\% rounded) | Observed number in proteins |  | Observed frequency (\%) in proteins |  | Observed frequency/Expected frequency |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Prokaryotes | Eukaryotes | Prokaryotes | Eukaryotes | Prokaryotes | Eukaryotes |
| Ala, A, Alanine (4/61) | 6.56 | 287360 | 535229 | 9.44 | 7.12 | 1.44 | 1.08 |
| Arg, R, Arginine (6/61) | 9.84 | 164498 | 377123 | 5.40 | 5.01 | 0.55 | 0.51 |
| Asn, N, Asparagine (2/61) | 3.28 | 127874 | 337816 | 4.20 | 4.49 | 1.28 | 1.37 |
| Asp, D, Aspartic acid (2/61) | 3.28 | 170363 | 392502 | 5.60 | 5.22 | 1.71 | 1.59 |
| Cys, C, Cysteine (2/61) | 3.28 | 30213 | 151439 | 0.99 | 2.01 | 0.30 | 0.61 |
| Gln, Q, Glutamine (2/61) | 3.28 | 119579 | 312059 | 3.93 | 4.15 | 1.20 | 1.26 |
| Glu, E, Glutamic acid (2/61) | 3.28 | 190544 | 488770 | 6.26 | 6.50 | 1.91 | 1.98 |
| Gly, G, Glycine (4/61) | 6.56 | 234665 | 515987 | 7.71 | 6.86 | 1.18 | 1.05 |
| His, H, Histidine (2/61) | 3.28 | 63907 | 175381 | 2.10 | 2.33 | 0.64 | 0.71 |
| Ile, I, Isoleucine (3/61) | 4.92 | 180442 | 402872 | 5.93 | 5.36 | 1.20 | 1.09 |
| Leu, L, Leucine (6/61) | 9.84 | 288009 | 690767 | 9.46 | 9.18 | 0.96 | 0.93 |
| Lys, K, Lysine (2/61) | 3.28 | 158728 | 466467 | 5.21 | 6.20 | 1.59 | 1.89 |
| M et, M , M ethionine (1/61) | 1.64 | 74134 | 173337 | 2.44 | 2.30 | 1.48 | 1.41 |
| Phe, F, Phenylalanine (2/61) | 3.28 | 114989 | 311375 | 3.78 | 4.14 | 1.15 | 1.26 |
| Pro, P, Proline (4/61) | 6.56 | 132643 | 390457 | 4.36 | 5.19 | 0.66 | 0.79 |
| Ser, S, Serine (6/61) | 9.84 | 183680 | 568759 | 6.03 | 7.56 | 0.61 | 0.77 |
| Thr, T, Threonine (4/61) | 6.56 | 173058 | 424350 | 5.69 | 5.64 | 0.87 | 0.86 |
| Trp, W, Tryptophan (1/61) | 1.64 | 38564 | 92793 | 1.27 | 1.23 | 0.77 | 0.75 |
| Tyr, Y, Tyrosine (2/61) | 3.28 | 94076 | 238524 | 3.09 | 3.17 | 0.94 | 0.97 |
| Val, V, Valine (4/61) | 6.56 | 216702 | 475037 | 7.12 | 6.32 | 1.09 | 0.96 |

nucleotides involved in frameshifting. The window of $X_{0}$ could be connected with this sequence.

In the -1 slipping process of prokaryotes, a secondary structure, usually a hairpin, and a Shine- D algarno site (Shine and Dalgarno, 1974) are often involved in frameshifting (V ögele et al., 1991; Larsen et al., 1994). The Shine-D algarno sites occur on average 14 nucleotides upstream ( -14 ) of the frameshift trinucleotides (L arsen et al., 1994), and the hairpin, seven nucleotides downstream $(+7)$ of the frameshift trinucleotides (Vögele et al., 1991). If the spacing between the Shine- Dalgarno site and the frameshift trinucleotide is reduced to seven nucleotides then +1 frameshift may occur (Larsen et al., 1994). N otes: the Shine-Dalgarno sites are observed six nucleotides upstream $(-6)$ of the frameshift trinucleotides in the +1 slipping process of prokaryotes (Weiss et al., 1988); a potential pseudoknot occurs five nucleotides downstream $(+5)$ of the frameshift trinucleotide in the +1 slipping process of the ornithine decarboxylase antizyme gene of eukaryotes (M atsufuji et al., 1995). The window of $X_{0}$ could be involved in these different sequences of critical lengths (Larsen et al., 1994).
The window of $X_{0}$ can also be considered according to the codon-anticodon pairing process as $X_{0}$ is a complementary circular code. The codon-anticodon pairing is classically based on three nucleotides. However, different models of extended codon-anticodon pairing have been proposed.
In the primitive translation RRY model proposed by Crick et al. (1976), the primitive tR N A s have an anticodon involving seven conserved nucleotides (two nucleotides on both sides of the anticodon in actual tRNAs; Barrell and Clark, 1974) in two alternative stacking configurations. The aminoacyl-tRNA adopts the $5^{\prime}$ stacked configuration with the five base anticodon $Y Y a_{1} a_{2} a_{3}, a_{1}$ being the wobble base (Woese, 1970). On the other hand, the peptidyl-tR NA adopts the $3^{\prime}$ stacked configuration with the five base anticodon $\mathrm{a}_{1} \mathrm{a}_{2} \mathrm{a}_{3} \mathrm{R} N$ (Fuller and Hodgson, 1967). A stacked configuration pairs with a five base codon. Note: a primitive codon-anticodon pairing involving only three bases such as an actual one without ribosome is not stable enough
for codon translation (G rosjean et al., 1978). The flip mechanism (detailed below for the RNY model) between these two stacked configurations leads to primitive protein genes consisting of a series of RRY codons (RRY model) (Crick et al., 1976). In the primitive translation RNY model proposed by Eigen and Schuster (1978), these biological concepts are considered with an anticodon involving only five conserved nucleotides (one nucleotide on both sides of the anticodon in actual tRNAs). Therefore, the aminoacyl-tR NA with the $5^{\prime}$ stacked configuration has the four base anticodon $Y a_{1} a_{2} a_{3}\left(3^{\prime} a_{3} a_{2} a_{1} Y\right)$ pairing with the four base codon $\mathrm{C}_{1} \mathrm{C}_{2} \mathrm{C}_{3} R$. The peptidyl-tR NA with the $3^{\prime}$ stacked configuration has the four base anticodon $a_{1} a_{2} a_{3} R$ ( $3^{\prime} R a_{3} a_{2} a_{1}$ ) pairing with the four base codon $\mathrm{YC}_{1} \mathrm{C}_{2} \mathrm{C}_{3}$ ( Fig . 6). The flip mechanism between these two stacked configurations keeps the three anticodon bases $3^{\prime} a_{3} a_{2} a_{1}$ paired with the three codon bases $\mathrm{C}_{1} \mathrm{C}_{2} \mathrm{C}_{3}$ for codon translation. The sequence $3^{\prime} R a_{3} a_{2} a_{1} a_{3} a_{2} a_{1} Y$ of the peptidyl and aminoacyl tRNAs paired with mRNA during the flip mechanism leads to primitive protein coding genes constituted of a series of RNY codons (Fig. 6). Thus, the window $3^{\prime} R a_{3} a_{2} a_{1} a_{3} a_{2} a_{1} Y$ of seven nucleotides with a shift of three bases according to a flip mechanism allows to generate the $\mathrm{C}^{3}$ code RNY on the 2-letter alphabet $\{\mathrm{R}, \mathrm{Y}\}$. A four base codon-anticodon pairing is also observed with the actual shifty tRNAs which are particular tRNAs involved in the translational frameshift sites (R iddle and Carbon, 1973; Beremand and Blumenthal, 1979; Tuohy et al., 1992). A biological model, such as the flip mechanism, for generating the $C^{3}$ code $\mathrm{X}_{0}$ on the 4 -letter alphabet $\{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$ remains to be found.

Finally, the window of $X_{0}$ may be related to the acceptor stem nucleotides for the correct recognition of the amino acid that the cognate aminoa-cyl-tRNA synthetase attaches to the tRNA 3'-terminus (Schimmel et al., 1993; Hartman, 1995; Saks and Sampson, 1995).
As the circular code $X_{0}$ is self-complementary, the two paired frames 0 in the two DNA double helix may simultaneously code for amino acids without the start codon (Fig. 7). Furthermore, as $X_{1}$ and $X_{2}$ are also circular codes and complemen-


```
Anticodon: 3' R 3 2 1 3 2 1 Y 5'
        R 
            R 
                R 
                                    R
                                    R R 3 2 1 1 3 3 2 1 1 Y
                                    R 
                                    R 
                                    R
Codon pattern: 5', Y R N Y R N Y R N Y RN YR N Y RN Y R . 3'
```

Fig. 6. The primitive translation R NY model of Eigen and Schuster (1978).
tary to each other and as a stop codon is never complementary to another stop codon, several frames among the six frames in the two DNA double helix may simultaneously code for amino acids without start codon, leading to an optimal information storage. For example, the insertion sequence IS1, i.e. a transposable element in bacteria, includes six potential reading frames with two genes encoded on the same strand of IS1 from partially overlapping reading frames (M achida et al., 1984). Overlapping genes on the same strand and genes on both DNA strands are classically found in the genomes of small size for maximizing the coding function, in particular in the mitochondrial genome, e.g. ATPase subunits six and eight, cytochrome oxidase subunit two and an adjacent ORF, etc. (Gray, 1992), and in the viral genome (Ziff, 1980). The concept of proteins coded by complementary strands has also been investigated for properties between amino acids and codons and for rules of the two complementary strands of DNA (Zull and Smith, 1990; K onecny et al., 1993; Béland and Allen, 1994; K onecny et al., 1995).
4.6. Conclusion summarizing the results

Based on a complete statistical analysis with 12288 autocorrelation functions, three subsets of 20 trinucleotides (by excluding A A A, CCC, G G G
and TTT) are identified in the three frames of protein genes of both prokaryotes and eukaryotes: $X_{0}$ in frame 0 and, $X_{1}$ and $X_{2}$ in the shifted frames 1 and 2 , respectively ( $X_{0}, X_{1}$ and $X_{2}$ being defined in Table 3(a)). U nexpectedly, $X_{0}, X_{1}$ and $X_{2}$ are maximal circular codes on the alphabet $\{A$, C, G, T\}, a concept suggested by Crick et al. (1957), then abandoned. These three circular codes have several relations between them. $X_{0}$ is self-complementary and, $X_{1}$ and $X_{2}$ are complementary to each other. $X_{0}$ generates $X_{1}$ by one circular permutation and $X_{2}$ by another circular permutation. Therefore, $X_{0}$ is a complementary Circular code with two permutated Circular codes (called $C^{3}$ code). The $C^{3}$ code $X_{0}$ has several properties: automatic retrieval of the frame after 13 nucleotides, rarity ( $6 \times 10^{-8}$ ) and concatenation properties (high frequency of misplaced trinucleotides in the shifted frames, maximum length of the minimal window to automatically retrieve the frame and the occurrence of the four types of nucleotides in the three trinucleotide sites). The different properties of the $C^{3}$ code $X_{0}$ as well as the few biological consequences analyzed here, in particular the possibility that several trinucleotides of $X_{0}$ may code for the same AA and the strong correlation between the usage of the trinucleotides of $X_{0}$ in protein genes and the amino acid frequencies in proteins, suggest that this code could have had a function in gene evolution and


Fig. 7. The self-complementary circular code $X_{0}$ allows the two paired frames 0 (reading frames) to code simultaneously for amino acids without a start codon.
that the primitive alphabet could have been $\{\mathrm{A}$, $\mathrm{C}, \mathrm{G}, \mathrm{T}\}$ rather than $\{\mathrm{R}, \mathrm{Y}\}$. An evolutionary model of the $C^{3}$ code $X_{0}$ based on substitutions is currently being investigated.

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